A Mixed-Integer Mathematical Model for Exam Timetabling: A Case Study at Fatih University Vocational School

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Introduction

In this paper, a mathematical model for exam timetabling is developed to schedule the final exams of Fatih University Vocational School (FUVS). Computations are carried out using Xpress MP Optimization Software. The results are significantly better than the existing manual approach implemented at FUVS in terms of conflict elimination and classroom usage efficiency.

Timetabling is a difficult (NP-complete) problem (Even et al., 1976). A general exam timetabling problem consists of assigning exams for different sections of offered classes to a limited number of time slots while specifying the classrooms for each assigned exam in a conflict-free fashion. The exams need to be scheduled while satisfying a set of constraints. These constraints are twofold: Hard constraints that cannot be violated in any circumstances, and soft constraints which should be satisfied as much as is possible (Qu and Burke, 2005).

To solve exam timetabling problems, many analytical and heuristic algorithms have been proposed, since no specific method can be applied universally due to the specifics of the individual problems. There are integer programming approaches for the solution of timetabling problems, as well as column generation, graph coloring algorithms and heuristics such as Tabu Search. A comprehensive survey on the literature of exam timetabling can be found in Burke and Petrovic (2002) and Burke et al. (1996).

Problem Definition and Mathematical Model

The main characteristics of the specific problem are as follows:

- Some courses have more than one section. Each section has its own final exam. So, more than one exam can be assigned to a specific course.
- Number of time slots: Currently, final exam period in FUVC is 13 days. Every day has four time slots. That is, 52 time slots are considered.
- A student can take at most two exams a day, and these exams can not appear in consecutive time slots.

Notations and Assumptions

Let n represent the exam day ($n=1,2,3,\ldots,13$).

There are four time slots in a given day specified as follows: 9:00-11:00, 11:00-13:00, 13:00-15:00, and 15:00-17:00. Hence there are 52 time slots.

Let T be the number of timeslots; $t=1,2,3,\ldots,T$.

Let E be the number of exams to be scheduled; $e=1,2,3,\ldots,E$.

Let S be the number of students; $s=1,2,3,\ldots,S$.

Let C be the number of classrooms; $c=1,2,3,\ldots,C$.

Let P_c be the capacity of classroom c ; $c=1,2,3,\ldots,C$.

Let W_e be the number of students taking a exam e ; $e=1,2,3,\ldots,E$.

Let $COMMON_{E\times E}$ be a matrix which includes the number of students taking two exams at a time.

Let $TAKEN_{S\times E}$ be a boolean matrix that represents courses taken by students. Courses taken by a student is represented as "1", others shown as "0".

Let $X_{E\times T}$ be a binary variable that takes the value "1" if the exam e is scheduled in time slot t and "0" otherwise.

That is:

$$x_{et} = \begin{cases} 1 & \text{If exam } e \text{ is assigned to time period of } t \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The developed mathematical model is:

$$\text{Min Total Conflict}=\sum_{d=1}^{E}\sum_{e=1}^{E}\sum_{t=1}^{T}(x_{et}\times x_{dt}\times COMMON_{ed}),\ d<e \tag{2}$$

Subject to:

$$\sum_{t=1}^{T}x_{et}=1 \text{ for each exam } e. \tag{3}$$

$$\sum_{e=1}^{E}W_e\cdot x_{et}\le\sum_{c=1}^{C}P_c \text{ for each time slot } t \tag{4}$$

$$\sum_{t=1}^{2}\sum_{e=1}^{E}TAKEN_{se}\cdot x_{et}\le 1 \text{ for each student } s \text{ and for each day } n. \tag{5}$$

$$\sum_{t=2}^{3}\sum_{e=1}^{E}TAKEN_{se}\cdot x_{et}\le 1 \text{ for each student } s \text{ and for each day } n. \tag{6}$$

$$\sum_{t=3}^{4}\sum_{e=1}^{E}TAKEN_{se}\cdot x_{et}\le 1 \text{ for each student } s \text{ and for each day } n. \tag{7}$$

The objective function (2) is minimizing the number of conflicts. Since the COMMON matrix is symmetric for this problem, we only need to define the pairs of exams d and e with $d < e$. If number of total conflict is zero we get the optimum solution.

The constraint (3) indicates that every exam should be scheduled exactly once. The constraint (4) implies that the number of students taking the exams scheduled to a specific time slot cannot exceed the total available classroom capacity. The last set of constraints (5,6 and 7) indicate that a student can take at most two exams a day, and these exams can not appear in consecutive time slots.

We implemented the exam timetabling model stated above using Xpress MP Optimization Software (Christelle G, 2000) on a Pentium IV 2.6 Ghz PC. It solves the problem using dual simplex method in 1580 iterations. Typical run time on the real data set from FUVS (217 exams, 617 students) equates to about 10 seconds. As a result, a conflict-free examination timetable has been found.

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