An Approximation Algorithm for the Traveling Tournament Problem*

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1 Introduction

In this abstract, we deal with the traveling tournament problem (TTP), which is a well-known benchmark problem established by Easton, Nemhauser and Trick [2]. We consider the case that both the number of consecutive away games and that of consecutive home games are at most three. We propose a lower bound of the optimal value of TTP, and construct an approximation algorithm yielding a feasible solution whose approximation ratio is less than $2+(9/4)/(n-1)$ where n is the number of teams.

Various studies on TTP have been appeared in recent years [2, 6]. Most of the best upper bounds for TTP are obtained by metaheuristic algorithms [1, 7, 8]. To the best of our knowledge, our result is the first approximation algorithm with a constant approximation ratio, which is less than 2+3/4.

2 Problem

In this section, we introduce some terminology and then define TTP. For more discussions on TTP and its variations, see [4, 5].

We are given a set of teams $T = \{1, 2, \ldots, n\}$ where $n \geq 4$ and even, and each team has its home venue. A game is specified by an ordered pair of teams. A double round-robin tournament is a set of games in which every team plays every other team once at its home venue and once at away (i.e., at the venue of the opponent); hence, exactly $2(n-1)$ slots are required to complete a double round-robin tournament.

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Each team stays its home venue before a tournament, and then travels to play its games at the chosen venues. After a tournament, each team goes back to its home venue (if necessary). We note that, when a team plays two consecutive away games, the team goes directly from the venue of the first opponent to the other without returning to its home venue.

For any pair of teams $i, j \in T$, $d_{ij} \geq 0$ denotes the distance between i's venue and j's venue. Throughout this abstract we assume that triangle inequalities ($d_{ij} + d_{jk} \geq d_{ik}$) and $d_{ii} = 0$ hold for any $i, j, k \in T$.

The traveling tournament problem is defined as follows.

Traveling Tournament Problem (TTP) [2]
Input: a set of teams T and a distance matrix $D = (d_{ij})$, indexed by T.
Output: a double round-robin tournament S of n teams such that

1. no team plays more than three consecutive away games;
2. no team plays more than three consecutive home games;
3. teams i at j immediately followed by j at i is prohibited (no repeaters);
4. the total distance traveled by the teams is minimized.

In the rest of this abstract, a double round-robin tournament satisfying the above conditions 1–3 (1–4) is called a *feasible* (*optimal*, respectively) tournament.

3 Lower Bound

In this section, we propose a new lower bound for TTP. We denote the sum total of ordered pair of distances between venues by Δ, i.e., $\Delta \stackrel{\text{def.}}{=} \sum_{\forall (i,j) \in T^2, i \neq j} d_{ij}$. In addition, to simply show the status of before and after a tournament, we introduce two artificial slots 0 and $2n-1$, and assume that each team is at home in these slots.

The following theorem provides a lower bound of the optimal value of TTP.

Theorem 1. *The optimal value z^* of TTP satisfies that $z^* \geq (2/3)\Delta$.*

Proof.

Let S^* be an optimal tournament of a given instance of TTP.

Suppose that team i plays three consecutive away games at teams j_1, j_2, then j_3. The total traveling distance of team i corresponding these games, denoted by $d(i; j_1, j_2, j_3)$, is $d_{ij_1} + d_{j_1 j_2} + d_{j_2 j_3} + d_{j_3 i}$. From the triangle inequality, we have $d(i; j_1, j_2, j_3) = d_{ij_1} + d_{j_1 j_2} + d_{j_2 j_3} + d_{j_3 i} \geq d_{ij_1} + d_{j_1 j_2} + d_{j_2 i} \geq d_{ij_1} + d_{j_1 i}$. Similarly, the following inequalities hold: $d(i; j_1, j_2, j_3) \geq d_{ij_2} + d_{j_2 i}$ and $d(i; j_1, j_2, j_3) \geq d_{ij_3} + d_{j_3 i}$. Hence, $d(i; j_1, j_2, j_3) \geq (d_{ij_1} + d_{j_1 i} + d_{ij_2} + d_{j_2 i} + d_{ij_3} + d_{j_3 i})/3$.

Next, consider the case that team i is at home in a particular slot, plays two consecutive away games at teams j_4 and j_5, then goes back to the home of i. The corresponding distance $d(i; j_4, j_5)$ is $d_{ij_4} + d_{j_4 j_5} + d_{j_5 i}$. We can easily show that $d(i; j_4, j_5) \geq (d_{ij_4} + d_{j_4 i} + d_{ij_5} + d_{j_5 i})/2 \geq (d_{ij_4} + d_{j_4 i} + d_{ij_5} + d_{j_5 i})/3$.

Finally, consider the case that team i is at home in a particular slot, plays an away game at team j_6, then goes back to its home. For the corresponding distance $d(i; j_6)$, we have $d(i; j_6) = d_{ij_6} + d_{j_6 i} \geq (d_{ij_6} + d_{j_6 i})/3$.

From the above, in the tournament S^* the traveling distance of team i is at least $(1/3)\sum_{j \in T\setminus\{i\}}(d_{ij}+d_{ji})$. Therefore we have $z^* \geq (1/3)\sum_{i\in T}\sum_{j\in T\setminus\{i\}}(d_{ij}+d_{ji}) = (2/3)\Delta$. □

4 Algorithm

In this section, for constructing good feasible tournaments, we propose a randomized algorithm based on the Modified Circle Method (MCM), which was proposed in [3] for the constant distance traveling tournament problem. (We omit the detail of MCM here.)

When a team plays at away in at least one of slots s and $s+1$, we say that the team has a *move* between these slots. On the number of moves in a feasible tournament, we have the following theorem.

Theorem 2. [3] *The algorithm MCM produces a feasible tournament S in which the number of moves, denoted by $M(S)$, satisfies that*

$$M(S) = \begin{cases} (4/3)n^2 - (2/3)n - 1 & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (1/2)n - 4/3 & (n \equiv 1 \mod 3), \\ (4/3)n^2 + (1/6)n - 5/3 & (n \equiv 2 \mod 3). \end{cases}$$

Here we note that MCM runs in $\mathrm{O}(n^2)$. Now we propose a simple randomized algorithm.

Algorithm 1

Step 1: Construct a feasible tournament S by MCM.

Step 2: Randomly permute the names of teams in S.

Then, we have the following theorem.

Theorem 3. *The approximation ratio of Algorithm 1 is bounded by*

$$2 + (9/4)/(n-1).$$

Proof. We denote the distance of a tournament obtained by Algorithm 1 by a random variable Z. The random permutation of names of teams implies that each move in tournament S obtained by MCM is assigned to a fixed pair of (mutually distinct) team venues with probability $1/(n^2-n)$. Thus, the expectation of the distance of a move is $\Delta/(n^2-n)$. Since MCM outputs a feasible tournament whose number of moves is less than $(4/3)n^2 + (1/6)n$, we have

$$\mathrm{E}[Z] < \big((4/3)n^2 + (1/6)n\big)\, \Delta/(n^2-n).$$

From the above, the approximation ratio of Algorithm 1 is strictly bounded by

$$\frac{\big((4/3)n^2 + (1/6)n\big)\, \Delta/(n^2-n)}{z^*} \leq \frac{\big((4/3)n^2 + (1/6)n\big)\, \Delta/(n^2-n)}{(2/3)\Delta} = (3/2)\frac{(4/3)n^2 + (1/6)n}{n^2-n} = 2 + \frac{9/4}{n-1}. \quad \square$$

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