An Approximation Algorithm for the Unconstrained Traveling Tournament Problem*

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1 The Unconstrained Traveling Tournament Problem

A deterministic 3-approximation algorithm is proposed for the unconstrained traveling tournament problem, which is a variant of the traveling tournament problem. For the unconstrained traveling tournament problem, this is the first proposal of an approximation algorithm with a constant approximation ratio. In addition, the proposed algorithm yields a solution that meets both the no-repeater and mirrored constraints.

In the field of tournament timetabling, the traveling tournament problem (TTP) is a well-known benchmark problem established by Easton, Nemhauser, and Trick [2]. The present paper considers the unconstrained traveling tournament problem (UTTP), which is a variant of the TTP. In the following, some terminology and the TTP are introduced. The UTTP is then defined at the end of this section.

Given a set $T = \{0, 1, \ldots, n-1\}$ of n teams, where $n \geq 4$ and is even, a game is specified by an ordered pair of teams. Each team in T has its home venue. A double round-robin tournament is a set of games in which every team plays every other team once at its home venue and once in an away game (i.e., at the venue of the opponent). Consequently, $2(n-1)$ slots are necessary to complete a double round-robin tournament.

Each team stays at its home venue before a tournament and then travels to play its games at the chosen venues. After a tournament, each team returns to its home venue if the last game is played as an away game. When a team plays two consecutive away games, the team goes directly from the venue of the first opponent to the venue of another opponent without returning to its home venue.

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For any pair of teams $i, j \in T$, $d_{ij} \geq 0$ denotes the distance between the home venues of i and j. Throughout the present paper, we assume that triangle inequality ($d_{ij} + d_{jk} \geq d_{ik}$), symmetry ($d_{ij} = d_{ji}$), and $d_{ii} = 0$ hold for any $i, j, k \in T$.

Denote the distance matrix (d_{ij}) by D. Given a constant (positive integer) $u \geq 3$, the traveling tournament problem [2] is defined as follows.

Traveling Tournament Problem (TTP(u))
Input: A set of teams T and a distance matrix $D = (d_{ij})$.
Output: A double round-robin schedule of n teams such that
C1. No team plays more than u consecutive away games,
C2. No team plays more than u consecutive home games,
C3. Game i at j immediately followed by game j at i is prohibited,
C4. The total distance traveled by the teams is minimized.

Constraints C1 and C2 are referred to as the *atmost* constraints, and Constraint C3 is referred to as the *no-repeater* constraint.

Various studies on the TTP have been conducted in recent years (see [4] for detail), and most of these studies considered TTP(3) [5]. Most of the best upper bounds of TTP instances are obtained using metaheuristic algorithms. On the other hand, little research on approximation algorithms has been conducted for the TTP. Recently, Miyashiro, Matsui, and Imahori [3] proposed a $(2 + O(1/n))$-approximation algorithm for TTP(3). In addition, Yamaguchi, Imahori, Miyashiro, and Matsui [6] proposed an approximation algorithm for TTP(u), where $u \ll n$. For TTP(3), the approximation ratio of [6] is better than that of [3].

The unconstrained traveling tournament problem (UTTP) is a variant of the TTP, in which Constraints C1 through C3 are ignored. In other words, the UTTP is equivalent to TTP($n-1$) without the no-repeater constraint. Although the UTTP is simpler than the TTP, no approximation algorithm for the UTTP has yet been proposed. The method proposed in [6] cannot be applied to the UTTP because the condition $u \ll n$ is necessary for the method. The method in [3], proposed for TTP(3), can be applied to the UTTP with a few modifications. However, this leads to a $((2/3)n + O(1))$-approximation algorithm, which is not a constant approximation ratio with regard to n.

In the present paper, we propose a deterministic 3-approximation algorithm for the UTTP. In addition, the solution obtained by the algorithm meets both the no-repeater and mirrored constraints, which are sometimes required in practice.

2 Approximation Algorithm

In this section, we describe the proposed approximation algorithm for the UTTP. A key concept of the algorithm is the use of the circle method and a shortest Hamilton cycle. The classical schedule obtained by the circle method satisfies the property such that the orders of opponents in almost all teams are very similar to a mutual cyclic order of teams. Roughly speaking, the proposed algorithm constructs a short Hamilton cycle passing all venues, and finds a permutation of teams such that the above cyclic order corresponds to the obtained Hamilton cycle.

For a vertex set $V = \{0, 1, \ldots, n-1\}$, let $G = (V, E)$ be a graph such that the distance of edge (i, j) is given by d_{ij} for any $i, j \in V$. First, we assign aliases $t_0, t_1, \ldots, t_{n-1}$ to teams $0, 1, \ldots, n-1$ as follows.

1. For each $v \in V$, compute $\sum_{v' \in V \setminus \{v\}} d_{vv'}$.
2. Let v^* be a vertex that attains $\min_{v \in V} \sum_{v' \in V \setminus \{v\}} d_{vv'}$, and designate the team corresponding to v^* as t_{n-1}.
3. Using the Christofides' 3/2-approximation algorithm for the traveling salesman problem with the triangle inequality [1], construct a Hamilton cycle on the complete graph induced by $V \setminus \{v^*\}$. For the obtained cycle $(v_0, v_1, \ldots, v_{n-2})$, denote the corresponding teams by $(t_0, t_1, \ldots, t_{n-2})$.

Next, we construct a single round-robin schedule. In the following, "schedule without HA-assignment" refers to a "round-robin schedule without the concepts of home game, away game, and venue." Denote the set of $n-1$ slots by $S = \{0, 1, \ldots, n-2\}$. A single round-robin schedule without HA-assignment is a matrix K of which (t, s) element, say $K(t, s)$, denotes the opponent of team t in slot s. Let K^* be a matrix defined by

$$K^*(t,s) = \begin{cases} t_{s-t \bmod n-1} & (t \neq n-1 \text{ and } s-t \neq t \ [\mathrm{mod}\ n-1]), \\ t_{n-1} & (t \neq n-1 \text{ and } s-t = t \ [\mathrm{mod}\ n-1]), \\ t_{s/2} & (t = n-1 \text{ and } s \text{ is even}), \\ t_{(s+n-1)/2} & (t = n-1 \text{ and } s \text{ is odd}). \end{cases}$$

Lemma 1. [6] *The matrix K^* is a single round-robin schedule without HA-assignment. In addition, K^* is essentially equivalent to the classical schedule obtained by the circle method.*

Then, by the mirroring procedure, form K^* into a double round-robin schedule without HA-assignment. Finally, we assign home and away so as to complete a double round-robin schedule as follows:

– for team $t \in \{t_0, t_1, \ldots, t_{n/2-1}\}$, let the games in slots $n+2t-1, n+2t, \ldots, n+2t+n-3$ [mod $2(n-1)$] be away games, and let the other games be home games.
– for team $t \in \{t_{n/2}, t_{n/2+1}, \ldots, t_{n-2}\}$, let the games in slots $2t-n+2, 2t-n+3, \ldots, 2t$ be away games, and let the other games be home games.
– for team t_{n-1}, let the games in slots $0, 1, \ldots, n-2$ be away games, and let the other games be home games.

The proposed double round-robin schedule, denoted by K^*_{DRR}, satisfies the no-repeater and mirrored constraints.

We now prove the above-mentioned algorithm is a 3-approximation algorithm for the UTTP. Designate the distance of a shortest Hamilton cycle on G as τ. In addition, let the distance of the cycle $(v_0, v_1, \ldots, v_{n-2})$ obtained above be τ'. Note that $\tau' \leq (3/2)\tau$.

Lemma 2. *The following propositions hold for G.*
(1) *For any path of two edges, its distance is bounded by τ.*
(2) *The distance of any Hamilton cycle is bounded by $n\tau/2$.*

In K^*_{DRR}, team t_{n-1} plays $n-1$ consecutive away games, and thus the distance by team t_{n-1} can be bounded by $n\tau/2$ from Lemma 2(2). In addition, analyzing the structure of the proposed schedule reveals the following lemma.

Lemma 3. *Let $l(i,j,k)$ be the distance of path (i,j,k) for $i,j,k \in V$. In K^*_{DRR}, the traveling distance of teams can be bounded by*

$$\begin{cases} \tau' + l(v_0, v^*, v_1) & (t = t_0), \\ \tau' + l(v_t, v^*, v_{t+1}) + l(v_{n-t-1}, v_t, v_{n-t-2}) & (t \in \{t_1, t_2, \ldots, t_{n/2-2}\}), \\ \tau' + l(v_{n/2-1}, v^*, v_{n/2-1}) & (t = t_{n/2-1}), \\ \tau' + l(v_{t-1}, v^*, v_t) & (t \in \{t_{n/2}, t_{n/2+1}, \ldots, t_{n-2}\}), \\ n\tau/2 & (t = t_{n-1}). \end{cases}$$

Although the following lemma is not obvious, we omit the proof due to space limitations.

Lemma 4. *Let v^* be a vertex that attains $\min_{v\in V} \sum_{v' \in V\setminus\{v\}} d_{vv'}$. Then, the following holds: $\sum_{v\in V\setminus\{v^*\}} d_{vv^*} \le n\tau/4$.*

Theorem 1. *The proposed algorithm is a 3-approximation algorithm for the UTTP.*
Proof. Let the distance of K^*_{DRR} be $d(K^*_{\mathrm{DRR}})$. From Lemmas 2 through 4, we have:

$$\begin{aligned} d(K^*_{\mathrm{DRR}}) &\le \tau'(n-1) + \sum_{t\in\{t_0,t_1,\ldots,t_{n-3}\}} l(v_t, v^*, v_{t+1}) + l(v_{n/2-1}, v^*, v_{n/2-1}) \\ &\quad + \sum_{t\in\{t_1,t_2,\ldots,t_{n/2-2}\}} l(v_{n-t-1}, v_t, v_{n-t-2}) + n\tau/2 \\ &\le (3/2)\tau(n-1) + \sum_{v\in V\setminus\{v^*\}} 2d_{vv^*} + \tau + \tau(n/2-2) + n\tau/2 \\ &\le (3/2)\tau(n-1) + 2(n\tau/4) + \tau + \tau(n/2-2) + n\tau/2 \\ &\le 3n\tau. \end{aligned}$$

Since $n\tau$ is a lower bound of the distance of any double round-robin schedule, this concludes the proof. □

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