The Relaxed Traveling Tournament Problem

Extended Abstract

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Abstract The Traveling Tournament Problem (TTP) is a sports scheduling problem that encapsulates two major aspects of some sports leagues: restrictions on acceptable home/away patterns and limits on travel distances. One major assumption in the TTP is that the schedule is compact: every team plays in every time slot. Some sports leagues have both pattern restrictions and distance limits but are not compact. In such schedules, one or more teams can have a bye in any time slot. We examine a generalization of the TTP where byes are possible.

Keywords Sports Scheduling · Integer Programming · Constraint Programming

Over the last twenty years, there has been increased interest in computational methods for creating sports schedules. This interest has been driven both by advances in the combinatorial structure of sports schedules and in the practical need for schedules by real sports leagues. There have been a number of recent surveys on the subject [5, 11,3] along with a recent annotated bibliography ([10]).

One path of research has revolved around the Traveling Tournament Problem (TTP). In the TTP, there are $2n$ teams, each with a home venue. The teams wish to play a double round robin tournament, whereby each team will play every other team twice, once at each team's home venue. This means that every team needs to play $2n-2$ games. There are $2n-2$ time slots in which to play these games, so every team plays in every time slot. Associated with a TTP instance is a distance matrix D where D_{ij} is the distance between the venue of team i and team j. Teams are assumed to begin and end the tournament at their home venue. If team i plays consecutive games at the venues of j and k, then i travels from its home venue to that of j then on to k before returning home to i's venue (and similarly for longer trips). The objective is to minimize the total travel of the teams subject to some requirements on the number

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of consecutive home (or road) games by each team. Those requirements can vary, but the canonical TTP requires that each team play no more than three consecutive home games or three consecutive road games.

The TTP was developed to abstract out the key issues in scheduling Major League Baseball, the United States professional baseball league. For that league, there are dozens of restrictions and requirements but the key issue was the tradeoff between distance traveled and home/away requirements. Since its introduction, the TTP has been the subject of numerous papers (see, for instance, [6, 2, 12, 9, 4, 14]) and is supported by an active website. Despite this interest, the TTP has proven to be a computational difficult challenge. For many years, the six-team instance NL6 was the largest instance solved to provable optimality. In 2008, NL8 was solved; NL10 was solved in late 2009 along with other ten team instances. This leaves twelve teams as the smallest unsolved instances, which still seems a remarkably small league size for such a simple problem description.

The goal of the TTP is to find a compact schedule: the number of time slots is equal to the number of games each team plays. This forces every team to play in every time slot. There are a number of leagues which are concerned with both home/away patterns and distance traveled but do not require compact schedules. Two significant examples in the United States are the National Basketball Association and the National Hockey League. Both leagues are economically significant, with yearly revenues of US$3.6 billion and US$2.8 billion respectively. If we examine the schedule for a team in each league, as shown in Figure 1, we can see a number of scheduling similarities.

The timetable at the top of Figure 1 is the schedule for the NBA's Cleveland Cavaliers for December 2009. In that schedule, home games are represented by darker background dates; away games have a white background. The timetable at the bottom of Figure 1 is for the NHL's Pittsburgh Penguins, with away games marked with an "@" symbol. For both leagues, the dates on which games are played vary by team. In fact, there are both NBA and NHL games every day of the months given. Over the course of a season, there is approximately one off day for every game played by a team (season lengths are 82 games per team for each league over approximately 160 days).

For both of these leagues, it is generally the case that teams with consecutive road games travel between the road cities, rather than returning home in between. This makes travel an important component of the schedule. For instance, the Penguins schedule begins the month with road games at Anaheim, Los Angeles and San Jose (all teams on the US west coast) before returning to the east coast to play at Boston and then returning home. This is a much better trip than Anaheim, Boston, Los Angeles and San Jose for a team based in eastern part of the United States, as Pittsburgh is.

These schedules lead to a natural generalization of the Traveling Tournament Problem which we call the Relaxed Traveling Tournament Problem (RTTP): instead of fixing the schedule length to be $2n-2$, let the schedule length be $2n-2+K$ for some integer $K \geq 0$. For a given K, we will denote the corresponding problem as the K-RTTP. For $K = 0$, the RTTP is just the TTP. For $K > 0$, each team has K slots in which it does not play. We call such a slot a bye for the team. There are many ways in which these byes could be counted. Initially, we will simply ignore byes when determining consecutive home or away games. So a home(H)/away(A)/bye(B) pattern of `HHBHAABBA` would be treated as having one three game home stand followed by a three game road trip. The advantage of this definition is that solutions for the TTP

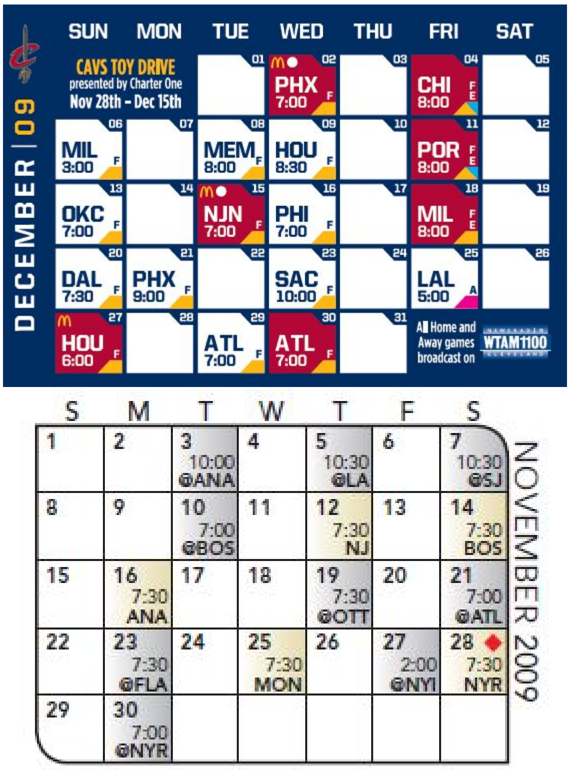

Fig. 1 NBA and NHL Schedules

are feasible for the K-RTTP for all $K \geq 0$ (in fact, k_1-RTTP are feasible for k_2-RTTP for $k_1 \leq k_2$).

For relatively small K this treatment of byes is reasonable. But for large K, simply ignoring the byes can lead to undesirable behavior whereby, for instances, a sequence like ABBBBBBBBBBA is treated like a two game road trip, when any real team would return home in the interim. For larger K (like $K = 2n - 2$, mimicking the NBA and NHL), we can put lower and upper bounds on the number of consecutive byes, or have other restrictions to have the patterns reflect playable schedules.

With this definition of K-RTTP, there are a number of interesting questions. Key to some of these is the idea of the Independent Lower Bound (ILB). For the TTP, the ILB is found by determining, for each team, the minimum distance that team must travel to visit all other teams, respecting limits on trip length. The ILB is then the sum of that value over all teams. Clearly the ILB is a lower bound for the TTP and for the K-TTP for all K. It is a reasonable conjecture that the ILB is tight for TTPs of at least a certain size. The work of Urrutia and Ribeiro [13] show this is not the case, even if there are no upper bounds on trip length and the distance between i and j is 1 for any $i \neq j$. Do byes help in this case? We conjecture that for sufficiently large K, the ILB is tight for the K-TTP, where K depends on n, but not on D. Even stronger, it may be that the ILB is tight for the 1-RTTP. While this may seem farfetched (can one bye per team be enough?), the work of [7] shows that for avoiding "breaks" (consecutive homes or aways), one bye per team is sufficient to reduce the number of breaks in a round robin schedule on $2n$ teams from $2n-2$ to zero. Perhaps one break is also enough to allow every team minimum travel. Our computational methods confirm this for NL4, though that is extremely slender evidence.

Key to exploration of this and other issues is the need for a computational approach for solving K-RTTPs. Over the last decade, there have been a number of approaches proposed to exactly solve the TTP (not counting many more heuristic approaches which are beyond the scope of this work). In [1], a number of alternative approaches were proposed, including generalizations of the well-known three phase approach (finding pattern sets, schedules, and game assignments) and trip formulations for the TTP. We have implemented integer and constraint programming versions to determine optimal schedules and are in the process of developing a system based on logic-based Benders decomposition [8]. With our current implementations, we can state the following:

1. The RTTP appears to be even harder than the TTP to solve to optimality
2. Even small K leads to interesting, difficult instances
3. Current techniques for the TTP can be generalized to the RTTP, though the generalizations are not trivial or straightforward.

In the full version, we will outline the generalizations and describe how they work computationally. We will also address issues of the structure of small K problems, and the relationship to the ILB.

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