Timetabling of sorting slots in a logistic warehouse

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Abstract We study a problem appearing at the end of a logistic stream in a warehouse and which concerns the timetabling of sorting slots necessary to accomodate the prepared orders before being dispatched. We consider a set of orders to be prepared on some preparation shops over a given time horizon. Each order is associated with the truck handling its transportation to its customer. A sorting slot is an accumulation area where processed orders await to be loaded on a truck. To a given truck, a known number of sorting slots is needed from the time the first order associated with the truck begins to be prepared, and this, until the known departure time of the truck. Since several orders associated with different trucks are processed simultaneously and since the number of sorting slots is limited, the timetabling of these resources is necessary to ensure that all orders can be processed over the considered time horizon without violating the resource constraint on sorting slots. In this talk, we describe the general industrial context of the problem and we formalize it. We state that some particular cases of the problem are polynomially solvable while the general problem is NP-complete. Then, we propose optimization methods to solve it.

Keywords warehouse management · sorting slots · scheduling · timetabling

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1 Introduction

Logistic warehouses are more and more complex installations: the flows regularly increase, the automation of systems become widespread, logistic processes are globalizing. To be competitive, warehouse management softwares (Warehouse Management Systems (WMS) and Warehouse Control Systems (WCS)) have to become a real help to make the best operational decisions.

Since the introduction of automation systems in production and distribution environments, many research studies on operational decision systems can be found in the literature. This subject has led to a large amount of research papers (see for example [1–4] for surveys on this area). In this paper, we study a sub-problem which appears at the end of a logistic stream in a warehouse and concerns the timetabling of sorting slots. To our knowledge, this problem has not been studied in the literature.

We consider a warehouse composed of several order preparation shops and a set of orders to be prepared on a given time horizon. Each order is composed of several order lines. Each order line corresponds to an amount of work to process on one of the preparation shops. Each order is associated with a transportation truck which is in charge to transport it to its final customer. It forces the subset of all orders associated with a given truck to be totally processed before the known departure of the truck.

Once an order has been prepared, it is dispatched to a sorting slot which is an accumulation area where the order waits to be loaded on the truck. The difficulty of the problem comes from a special resource constraint: to a given truck, a known number of sorting slots is needed from the time the first order associated with the truck begins to be prepared, and this, until the known departure of the truck. Since several orders associated with different trucks are prepared simultaneously and since the number of sorting slots is limited, the timetabling of the sorting slots becomes necessary. In other words, we have to establish the timetable of sorting slots, in aim to ensure that all orders can be processed over the considered time horizon without violating the resource constraint on the number of available sorting slots.

2 Problem definition

The problem in hand can be formalized in the following way. Let n be the number of trucks and let m be the number of preparation shops in the warehouse. A preparation shop is assimilated as a machine. To each truck i is associated a task J_i composed of m operations $\{o_{i1}, o_{i2}, \ldots, o_{im}\}$. The processing time p_{ij} of operation o_{ij} corresponds to the duration of the whole set of order lines which have to be processed by machine (preparation shop) j associated with truck i, *i.e.* to the total amount of work which has to be done by shop j for truck i. Each of the m machines has then to process n operations (one per task). Let d_i be the known date of the departure of truck i. In a feasible solution of the problem, each task J_i is considered being finished by time d_i, while the start-time s_i of J_i is to be determined such that all operations of J_i can be performed. The set of sorting slots is assimilated as a cumulative resource of capacity E, E being the number of sorting slots available in the warehouse. Thus, task J_i needs a given number n_i of sorting slots among E from s_i until d_i. Note that the period on which the resource constraint applies depends only on the start-time of the job, contrary to classical scheduling problems in which it depends on periods on which the job is really processed. The problem is to determine start times $\{s_1, \ldots, s_n\}$ in such a way that no more than E sorting slots can be used at a time, while the scheduling of operations is feasible.

Let s_{ij} and c_{ij} be respectively the variables representing the start time and the completion time of operation o_{ij} in a solution. In a feasible solution, each operation o_{ij} can start after the start of task J_i and has to be completed before the departure time of the associated truck: $s_{ij} \geq s_i$ and $c_{ij} \leq d_i$. Each machine is disjonctive and can process only one operation at a time. Note that, contrary to a scheduling shop problem (see *e.g.* [5]), the operations of a same task can be processed simultaneously (and, in fact, will be very often).
Two cases are considered:

- The non-preemptive case in which, once an operation o_{ij} begins to be processed, it has to be continued until its completion ($s_{ij} + p_{ij} = c_{ij}$). This implies that all order lines associated with a truck will be dealt sequentially.
- The preemptive case, in which interruptions in the processing of tasks are possible ($s_{ij} + p_{ij} \leq c_{ij}$). Such interruptions are useful to execute a part of another operation more urgent associated with another truck.

If variables $\{s_1, s_2, \ldots, s_n\}$ can be fixed in such a way that all the previous constraints hold, it follows that n_i sorting slots among E will be used from s_i to d_i for truck i.

In the remainder, we use also the following notation. Let $H = \max_{i \in \{1,\ldots,n\}} d_i$ be the time horizon of an instance of the problem. Let also $T = \{d_i / i \in N\} + \{0\} = \{t_0 = 0, \ldots, t_w = H\}$ be the set of departure dates, values t_i being indexed in non-decreasing order, *i.e.* $t_0 = 0 < t_1 < \ldots < t_w = H$.

3 Complexity study of the problem

The special preemptive case in which we have $\sum_{i=1}^{n} n_i \leq E$ is polynomially solvable. Indeed, the ressource constraint according to the number of sorting slots disapears in this case. Therefore, it leads to the resolution of m single-machine scheduling problems with deadlines $(1|\bar{d}_i|\_)$ which are polynomially solvable [6].

The other special case in which any $p_{ij} = 1$ is also polynomially solvable. Indeed, the operations can be scheduled in the same way on each machine. Thus the problem is reduced to another one with $m = 1$. It then remains to know if we can schedule operations without violating the resource constraint on sorting slots. For that, we iteratively schedule operations from H to 0 in decreasing order of departure times. When several operations have the same departure time, they are scheduled in decreasing order of the number of sorting slots. The process stops when all operations are scheduled or when the resource constraint on sorting slots does not hold (the instance is not feasible).

We state that the 3-PARTITION problem [6] can be reduced to the general preemptive case with $m = 1$. It follows that both the general preemptive and non-preemptive problems are strongly NP-complete.

4 Some methods to solve the problem

We propose optimization methods to solve both the preemptive and the non-preemptive versions of the problem. The proposed methods relies on the following proposition:

Proposition 1 *Let S be a feasible solution of an instance of the problem in which a job i is executed from s_i to d_i. Then S can be transformed in a feasible solution S' in which job i uses n_i sorting slots from $s'_i = \max\{t_k, t_k \in R \wedge t_k <= s_i\}$ and where the scheduling of operations is not modified.*

Thus, we can deduce the following dominance rule.

Theorem 1 *There exists at least one solution in which the start time of use of sorting slots take values in set T.*

This last proposition can be used in several ways. From a practical point of view, it offers a method to simplify the timetabling of the use of the sorting slots since they can be established by periods determined by the departure times of the different trucks. Of course, the additional use (a priori useless) of sorting slots by a job can be seen as additional flexibility in the preparation process. From a resolution point of view, this dominance rule is used to establish integer linear programmig formulations and constraint based scheduling methods [7] to solve in pratice the problems. Thus, we propose an ILP model and a constraint programming model to solve the non-preemptive case. Then, we propose a flow-based ILP model and a multiperiod ILP model to solve the preemptive case. These methods are experimentally compared from a computational point of view.

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