An exact decomposition approach for the optimal real-time train rescheduling problem

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Trains are running through a rail network trying to meet a predefined schedule, the *Official Timetable*, which specifies when each train enters and exists the stations on its route. When one or more trains deviate from the official timetable, new schedules and possibly new routes must be identified and implemented very quickly. Also, the new plan should minimize some measure of the delays.

In a first and very simplified picture, a rail network may be viewed as a set of stations connected by tracks. Each train follows a specific route in this network, namely an alternating sequence of stations and tracks. The trains run their routes trying to agree with the *production plan*, which specifies the movements (*routing*) and the times when a train should enter and leave the various segments of its route (*schedule*), including stations arrival and departure times.

In principle, the production plan ensures that no two trains will occupy simultaneously the same railway resource, or incompatible resources such as a platform in a station and the track to access it. In other words, a production plan is a *conflict free* schedule. The problem to design optimal production plans is of crucial relevance for railway operators. As pointed out in [11] *optimum resource allocation can make a difference between profit and loss for a railway transport company.* However, due to different causes the actual train timetables can deviate from the official ones, and potential conflicts in the use of resources may arise. As a consequence, re-routing and re-scheduling decisions must be taken in real-time. These decisions are still, in most cases, taken by human operators (*dispatchers*), and implemented by re-orienting switches and by controlling the signals status (i.e. setting signalling lights to green or to red), or even by telephone connections with the drivers. The dispatchers take their decisions trying to minimize delays, typically having in mind some ranking of the trains or simply following operating rules. So, what the dispatchers are actually doing, is solving an optimization problem (and of a very tough nature). We call this problem the *Real-time Traffic Control in Rail Systems* problem (*RTC*).

In short, the RTC problem amounts to establish *in real-time* for each controlled train a route and a schedule so that no conflicts occur with other trains and some

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function of the deviation from the official timetable is minimized. As such, the RTC problem falls into the class of *job-shop scheduling problems* where trains correspond to jobs and the occupation of a railway resource by a train is an operation. Two alternative classes of formulations have been extensively studied in the literature for job-shop scheduling problems and consequently also applied to train scheduling and routing problems, namely the *time indexed formulations* [10] and the *disjunctive formulations* [2].

In time indexed formulations (TI) the time horizon is discretized, and a binary variable is associated with every operation and every period in the time horizon. Conflicts between operations are prevented by simple packing constraints. Examples of applications of (TI) to train optimization can be found in [3], [4], [5], [6], [18]: actually the literature is much wider, and we refer to [8], [11] and [16] for extensive surveys. To our knowledge, basically all these works deal with the track allocation problem, which is solved off-line and where the feasible time periods associated with train routes are strongly limited by the tentative timetable. In contrast, in the RTC problem the actual arrival and departure times may differ substantially from the wanted ones. Consequently, the number of feasible time periods grows too large to be handled effectively by time-indexed formulation within the stringent times imposed by application, as extensively discussed in [13].

In disjunctive formulations, continuous variables are associated with the starting times of the operations, whereas a conflict is represented by a disjunctive precedence constraints, namely, a pair of standard precedence constraints at least one of which must be satisfied by any feasible schedule. The *disjunctive graph* ([1]), where disjunctions are represented by pairs of directed arcs, can be associated to any disjunctive formulation and exploited in solution algorithms. The disjunctive formulation associated can be easily transformed into a Mixed Integer Linear Program (MILP) by associating a binary variable with every pair of (potentially) conflicting operations and, for any such variables, a pair of *big-M precedence constraints* representing the original disjunction. These constraints contain a very large coefficient and they tend to weaken the overall formulation and this is mainly the reason why (TI) formulations were introduced.

The connection between railway traffic control problems, job-shop scheduling and corresponding disjunctive formulations was observed quite early in the literature. However, a systematic and comprehensive model able to capture all the relevant aspects of the RTC was described and studied only much later in the Ph.D. thesis by Alessandro Mascis [14] and further developed in [15]. In these works, the authors also introduce a generalization of the disjunctive graph that they call *alternative graph* but referred here simply as disjunctive graph. After these early works there has been a flourishing of papers representing the RTC by means of disjunctive formulations and exploiting the associated disjunctive graph. Recent examples can be found in [7], [8], [9], [17]. A comprehensive list of bibliographic references is out of our scope and again we refer to the above mentioned surveys. The great majority of these papers, however, only use the disjunctive formulation as a descriptive tool and resort to purely combinatorial heuristics to solve the corresponding RTC problems. The explicit use of the disjunctive formulation to compute bounds is quite rare, and typically limited to small or simplified instances. Examples are [13], which handles small-scale metro instances, and [17], which introduces several major simplifications, drastically reducing the instances size.

So, methods based on mathematical programming are rarely applied to solve real-life instances of the RTC problem: time-indexed formulations tend to be too large and often cannot even generate a solution within the time limit; big-M formulations tend to be too weak and they can fail to produce feasible solutions within the time limit.

In this presentation we introduce a new modelling approach to RTC and a solution methodology which allow to overcome some of the limitations of the standard big-M formulations and solve to optimality the corresponding big-M MILP within the stringent running times imposed by the application for a number of real-life instances in single-track railways. The methodology is based on a structured decomposition of the RTC into two sub-problems: the *Line Traffic Control Problem* (LTC) and the *Station Traffic Control Problem* (STC). The LTC amounts to establishing where potentially conflicting trains should meet along the network. When dealing with single-track lines, this may only happen in stations (or similar infrastructures). The STC problem is the problem of routing and scheduling trains in a station (in real-time). The LTC problem and the STC problem give raise to distinct sets of variables and constraints, which are then solved in a joint model by row and column generation.

The decomposition has two major advantages. First, the number of variables and big-M constraints is drastically reduced with respect to the standard big-M formulations. The second advantage is that we have some degrees of freedom in modelling the STC problem. Indeed, we will show that the (general) STC problem is NP-hard. However, in some cases of practical impact, simpler models can be considered, leading to polynomial cases: we will describe one such case, very common in practice. Actually, since the lines may contain quite different stations' layouts, different models can/must be applied simultaneously.

Interestingly, this decomposition resembles the normal practice of railway engineers to distinguish between *station tracks* and *line tracks* (see, e.g. Conte 2007) and of actually tackle the two problems separately.

This decomposition approach has been successfully applied in a rescheduling system operating a number of single and double track lines in Italy [12].

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