A real world personnel rostering problem with complex objectives

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We consider a practical personnel rostering problem faced by a public transportation company. The problem has some interesting objectives that we are not aware of having been treated before.

The problem consists of assigning shifts to a group of drivers over a time period. Each driver is to be assigned one shift on each day in the period. Except for shifts that represent a day off, the available shifts are unique; each can be assigned only on one specific day, and not to more than one driver. A number of day-off shifts are pre-assigned in the plan, yielding a fixed free day pattern for each driver. Shifts for the initial part of the period are also pre-assigned, providing a historical plan that must be sensibly continued.

Each shift has a start time, an end time and two durations: the work time and the driving time. Driver can express preferences as to which shifts they like or not, yielding a positive or negative preference value for each shift /driver pair.

The chief hard constraints are:

- Driver/ shift compatibility
- The accumulated work time for a driver may at no time deviate more than 40h from the nominal value

The objectives considered are:

- Maximize the preference values of the shift /driver assignments, fairly among the drivers
- Penalize large deviations of the accumulated work times from the nominal values at the end of the period
- Minimize the number of days off assigned in excess of the free day patterns
- Avoid many similar (by start and end time) consecutive shifts
- Avoid violations of the EU work and driving time regulations, plus rules set by local agreements

We solve the problem using local search. The following operators are used to modify the solution:

- Replace the shift assigned to a cell (driver/day pair) with an unassigned shift
- Swap the shift assigned on one day between two drivers
- Swap the shifts assigned on two distinct days between two drivers
- Swap the shifts assigned on a range of three or more consecutive days between two drivers

To establish an initial solution, we start with an empty solution, where all assignments not initially fixed are set to a day off. We make a few passes of applying the Replace operator in each cell, only accepting improving changes. We then move to the main search phase.

The main search is a focused tabu search. Each objective is responsible for identifying focal points, i.e. areas where the incumbent solution could be improved, and estimate the possible associated gain in objective value. In each iteration, the focal point with the highest estimated gain is selected. A neighborhood is generated by restricting the full set of solutions defined by the above operators, in two ways. Firstly, the solution must be modified in a region given by the selected focal point (e.g. a particular driver or day range). Secondly, the objective that generated the focal point must be improved locally in that region. This eliminates many solutions that need not be evaluated by all constraints and objectives. The best solution in the neighborhood is chosen as the new incumbent solution. A tabu criterion based on the recently modified cells is used to further reduce the neighborhood and to avoid cycling.

The major motivation for solving this problem is to increase driver satisfaction by allowing them to express their duty preferences and satisfying those preferences. Thus modeling the preference satisfaction objective correctly is vital to the implementation's success, and we present it in more detail.
There are several requirements for this objective:

- The basic effect is to maximize each driver's total preference for his assigned shifts
- If not all drivers can get their preferred shifts, the good assignments shall be distributed fairly among the drivers
- A driver should benefit from being flexible, that is, giving preferences that are easy to satisfy, as this also helps satisfying other drivers' preferences
- A driver should not be able to play the system, that is, benefit from expressing preferences that are not real

We propose the following objective formulation:
For each driver and day, define the driver's *luck* with the day's shift assignment as $l = P - E(P)$. P is the actual assignment's preference value, and $E(P) = \frac{1}{n}\sum_{i=1}^{n} p_i$ is the expected preference value, assuming that each compatible shift for that day is equally probable.
Let L_j be the sum of daily luck values over the period for driver j. Then we wish to maximize each value in $\{L_j\}$, while also minimizing inequality among the values.
Let $a_1 \dots a_m$ be a non-decreasing sequence of positive numbers with $a_1 < a_m$, and let $K_1 \dots K_m$ be the values in $\{L_j\}$ sorted from high to low. The total preference to be maximized is defined as $V = \sum_{j=1}^{m} a_j K_j$.
We argue that this objective definition satisfies the requirements above.

Another objective of interest penalizes violations of the EU work and driving time regulations. This objective is expensive to evaluate due to the rules' complexity. In each week, a driver's plan must contain a weekly rest period (from one shift's end time to the start time of the next) of sufficient length. Several aspects of the rules make the choices of rest period in different weeks interdependent:

- There is a limit on the maximum time between rest periods in consecutive weeks
- A rest that spans the border between two weeks can be assigned to either week, but not both
- A rest can be shorter than the normal length, but only if the next rest has at least normal length, and providing that the reduction is compensated over the next three weeks

Testing whether a solution has a legal weekly rest assignment requires us to perform a costly tree search. This leads us to putting this objective among the ones to be evaluated last.

We present results on real life problem instances covering up to 46 drivers over an 8 week period. User reports on the quality of the solutions produced during pilot tests are favorable.