A Column Generation Approach for Solving the Patient Admission Scheduling Problem

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Abstract The Patient Admission Scheduling Problem (PAS) is the problem of assigning patients to rooms during their stays in the hospital over a predefined planning period. Each room has a number of beds, which is the capacity of the room. A set of hard constraints determine whether or not a patient can stay in a room. For each patient-room combination a penalty is given for having the patient in the room. The penalty measures the inconvenience of being in the room for the given patient. Transferring patients between rooms during their stays is allowed, but this is penalized, as it is both inconvenient for the patient and takes time for the staff to organize. Finally the rooms are gender segregated, i.e. only patients of the same gender can stay in any room in the same time period. This constraint is considered a hard constraint. The problem is then to assign patients to rooms in their admission periods such that the total penalty is minimized and the hard constraints are satisfied.

PAS was proposed by Demeester et al. (2010) who solved it by a hybrid tabu search heuristic. Ceschia and Schaerf (2011) use simulated annealing to identify feasible upper bound solutions and they provide several lower bounds based on the assignment problem as well as linear programming relaxations of an integer programming model. Finally, a hyper-heuristic approach is described by Bilgin et al. (2011)

We present a Dantzig-Wolfe decomposition of PAS into a set-partitioning problem as the master problem and a set of room scheduling problems as the pricing problems. The set-partitioning problem has columns corresponding to feasible schedules for the rooms and it has two types of rows: a row for each patient-time combination stating that the patient has to be in a room in the time period and a

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row for each room type stating that we cannot use more than the available number of rooms of that type. The room scheduling problem for a specific room has to select a number of patients in each time period such that the number of patients is not greater than the capacity of the room and the genders of the patients chosen in each time period are identical. The room schedules have to be constructed such that the reduced cost coefficients of the schedules are minimized.

We solve the pricing problems by a series of greedy heuristics as well as a new exact dynamic programming based algorithm identifying the most negative reduced cost column for a specific room type. The latter has a stage for each time period and states composed of the cost of entering the state as well as the patients present in the room. Each stage has at most $\binom{n_t+Q}{Q}$ states, where n_t is the number of patients in period t and Q is the capacity of the room. This is exponential in Q but in our case Q never becomes more than 4. To reduce the number of states we apply several sufficient dominance criteria eliminating states which will never yield an optimal negative reduced cost column. We especially introduce pairwise patient dominance, which is used to prove state dominance. To further reduce the computation time we introduce preprocessing for the pricing problem based on shortest path calculations in two different acyclic graphs.

The master problem is severely degenerate, which makes it hard to solve the linear-programming relaxation directly by column generation. As a consequence, we apply dynamic constraint aggregation as proposed by Elhallaoui et al. (2005). This improves the performance of the column generation significantly.

We introduce several branching strategies to integerize the lower bound solution. The first is to branch on room types where a fractional number of rooms of the type is used. Next, if a room in a period has a fractional number of patients with one gender, then we branch on the gender, i.e. requiring the room to be either of this gender or not. Finally, if a patient is used fractionally in a time period for a room type, then we branch by either forcing the patient to be in the room at that time or prohibiting the patient from being in the room at that time.

The method is tested on benchmark instances described by Demeester et al. (2008) where we derive tighter lower bounds for several of the instances than previously reported. The computation times for identifying these lower bounds are in most cases significantly less than those presented by Ceschia and Schaerf (2011).

Keywords Patient Admission Scheduling · Decomposition · Dynamic Constraint Aggregation

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