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Models for the Shift Design Problem

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Abstract Rostering and staff scheduling problems often have a predefined set of shifts to which staff members can be allocated. These shifts are typically based on a small set of shift types, where each shift type is characterized by the period of the day the shift covers. The number of people assigned to a shift type reflects to some extent the demand for staff during the period it covers. We address the issue of whether or not we have the correct set of shift types given that we know the required staffing levels over a single day or a set of days.

The Shift Design Problem (SDP) is the problem of identifying the set of shift types prior to the solution of rostering or staff scheduling problems. This should be done in such a way that the demand for staff in each period of the day is matched as closely as possible, but with restrictions on the number of the staff used each day as well as the number of shift types used.

The SDP is a variant of the shift scheduling problem where – among other constraints – the number of shift types used is upper bounded. Shift scheduling has been considered by Dantzig (1954), where undercovering is prohibited and overcovering is free. Aykin (1996) extends this with breaks on the shifts and Mehrotra et al. (2000) develops a Branch-and-Price method for shift scheduling. The SDP is solved heuristically by Di Gaspero et al. (2013).

The motivation for the problem is based on the study by Lusby et al. (2012), who discuss a rostering problem for ground crew at airports. At an airport the required staffing level over a day is correlated with the arrival times of incoming flights and departure times of outgoing flights as well as the number of passengers on the flights. This staffing level demand can be forecast, yielding a demand

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curve for each day of the week. With the exception of holiday seasons, it can be observed that this demand is cyclic with a periodicity of one week. Hence we limit our attention to at most seven different demand curves. Based on these demand curves Lusby et al. (2012) show that it is better to let the rostering determine the number of persons used on each shift type than fixing this number prior to solving the rostering problem. However, the authors only consider a small number of pre-determined shift types; here we are interested in whether or not this combination of shift types is the best combination.

Let $\mathcal{D}$ be the set of days. Suppose that a day is discretized into periods $\mathcal{T} = \{1, \ldots, T\}$, each covering an interval of time, I_t, such that the day is partitioned by these intervals. Associated with each day $d \in \mathcal{D}$ and each period $t \in \mathcal{T}$ is a known required staffing level $\delta_{dt} \geq 0$. Additionally, a set of possible shifts $\mathcal{S}$ is given, and each shift $s \in \mathcal{S}$ covers a subset of the periods $\mathcal{P}_s \subseteq \mathcal{T}$. The staff demand for any period $t \in \mathcal{T}$ on a given day $d \in \mathcal{D}$ does not need to be strictly satisfied; however, for each unit of under cover (over cover) a penalty cost u_{dt} (o_{dt}) is incurred.

A solution to the SDP is hence a selection of a subset of shift types $\overline{\mathcal{S}} \subseteq \mathcal{S}$ and for each $s \in \overline{\mathcal{S}}$ an assignment of staff $n_{ds} \geq 0$ to each day $d \in \mathcal{D}$. This selection has to be done such that $|\overline{\mathcal{S}}| \leq K$ and $\sum_{s \in \overline{\mathcal{S}}} n_{ds} \leq N$ for each day $d \in \mathcal{D}$. That is, the number of shifts selected is no larger than a pre-specified number K, and no more than N staff members are assigned each day. From such a solution the amount of under cover and over cover in each period of each day can easily be determined. The constraint on the number of shift types used makes the problem difficult as a selection of shifts which enables a close match of the demand for one day may not be able to match the demand of another day well.

We suggest a mixed integer linear programming (MILP) model for the problem. The model uses two types of variables; one indicating whether or not a shift type is used and one counting the number of persons used on a given shift. These are coupled by big-M constraints – one for each shift type – making the resulting LP-relaxation weak. For $|\mathcal{D}| = 1$ the model can be solved to optimality using a commercial solver, while it becomes harder as $|\mathcal{D}|$ increases.

We discuss two approaches to solve the problem. The first approach is a column generation approach in which each column corresponds to a selection of shift types (together with their respective staffing levels) for each day. The master problem then makes a selection of the columns to match the demand on each day. The pricing problem disregards the demand covering constraints and thereby becomes a multidimensional cardinality constrained knapsack problem.

The MILP model exhibits a clear block angular structure where the only common constraint is the number of shift types used and each block corresponds to assigning a staffing level to shifts on a particular day. This lends itself to Benders decomposition where the master problem determines the set $\overline{\mathcal{S}}$, and the subproblems determine n_{ds} for each day $d \in \mathcal{D}$ and $s \in \overline{\mathcal{S}}$. The subproblems generate optimality cuts for the master problem.

We test and discuss our approach in relation to two airport cases. In this discussion we will point out the limitations as well as possible extensions of our models.

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