10th International Conference of the Practice and Theory of Automated Timetabling
PATAT 2014, 26-29 August 2014, York, United Kingdom

Optimal Duty Rostering for Toll Enforcement Inspectors

Elmar Swarat · **Guillaume Sagnol** ·
Thomas Schlechte

Abstract This abstract presents an Integer Programming based approach on optimal inspector rostering for the toll enforcement on German motorways.

Keywords Duty Rostering · Large Scale Integer Programming · Multi-Commodity Flow Problem

Mathematics Subject Classification (2000) 90B20 · 90C06

1 Abstract

We address the problem of planning optimal toll inspector tours on German motorways. Since 2005 a distance-based toll for all commercial trucks weighting 12 tonnes or above is set up on German motorways. A major part of the enforcement is conducted by tours of inspector teams on the toll network. Important enforcement goals are to take the spatial and temporal traffic distribution into account and to cover the complete network by unpredictable controls. Therefore, one task is to plan daily tours for the inspectors. In additon, a crew must be assigned to each tour, while each duty of a crew has to fit in a monthly (duty) roster. A feasible roster has to comply with a lot of different rules. Hence, a major challenge of our model is to optimize the rosters of the inspectors. This is the main focus of our presentation.

In other applications, e.g., in the railway or airline industry, the rostering is part of a multi-stage planning process. In particular, in duty scheduling tasks, e.g., timetabled trips, are covered by duties or pairings and afterwards rosters

This work was funded by the Federal Office for Goods Transport (BAG).

Elmar Swarat · Guillaume Sagnol · Thomas Schlechte
Zuse Institute Berlin, Takustr. 7, D-14195 Berlin
Tel.: +49-30-84185244
Fax: +49-30-84185269
E-mail: swarat@zib.de

are computed that cover all duties or pairings. In our setting a sequential planning of tours and crews is not appropriate according to the spatial distribution of the inspectors. Therefore, an integrated model of tour planning and duty rostering must be considered. We called this problem Toll Enforcement Problem (TEP). In [1] a case study presents the benefits of using our approach for the toll enforcement. Here, we extend our previous publications by a deeper look on the modeling and solving issues of the rostering part and by computational results that stem from production operation. The method can be transferred to other inspection or monitoring problems, where the availability of crews is an important issue for the planning problem.

Our approach is similar to the model, that Cappanera and Gallo [2] used to solve an airline crew rostering problem. Our model extends their approach in the respect that no activity has to be covered but there are coupling constraints that connect duties with tours. The base of our model is a directed graph $D = (V, A)$. The nodes $v \in V$ correspond to potential duties, that are defined as a triple $v = (d, b, e)$ of a day d, start time b, and end time e. Arcs $a = (v_1, v_2) \in A$ connect two duties $v_1 \in V$ and $v_2 \in V$, if v_2 is a feasible subsequent duty of v_1 according to legal rules. In addition, there are two nodes s und t indicating the beginning and end of the roster. Arcs $a = (s, v_i)$ connect s with all nodes v_i that might be the first duty of a roster. Analogously, arcs (v_i, t) connect potential last duties with t. Hence, a feasible duty roster corresponds to a s-t-path in D. We will discuss in detail, how several requirements can be modeled by this graph construction, like minimum rest times, days of leave or pre-assigned duties. The idea is to model as many constraints as possible as local decisions in our graph model.

The objective for the roster optimization is to minimize some artificial costs associated with unintended sequences of duties. Therefore, for each $a \in A$ artificial costs $c_a \geq 0$ are defined which have a non-zero value, if the current sequence of duties corresponds to *rotations*, i.e., changes in the duty starting time on two subsequent days. It is particularly known for rotations, where the following duty starts earlier, that they alter the human biorythms and affect the sleep. We model this optimization problem by a multi-commodity flow problem in D. For each inspector $m \in M$ we introduce binary flow variables $x_a^m \in \{0, 1\} \, \forall a \in A$. This results in the following Integer Program (IP):

$$\min \sum_{m \in M} \sum_{a \in A} c_a x_a^m \tag{1}$$

$$\sum_{v} x_{s,v}^m = 1, \qquad \forall m \in M, \tag{2}$$

$$\sum_{k} x_{v,k}^m - \sum_{u} x_{u,v}^m = 0, \qquad \forall v \in V, m \in M, \tag{3}$$

$$Rx \leq r, \tag{4}$$

$$x_a^m \in \{0, 1\}, \quad \forall a \in A, m \in M. \tag{5}$$

The objective function (1) minimizes the cost of the rosters. Constraints (2) and (3) represent the flow value and the flow conservation for the inspectors. Constraints (4) model additional requirements for feasible roster paths in D that we describe hereafter. In (5) the integrality constraints of the flow variables are given.

Each path consumes units of limited resources on its sequence arcs. In (4) this is coded by the matrix R and the limitation by the right hand side r. The resources deal with those requirements that can not be modeled locally in the graph. One example are *horizontal rules*. They involve single paths, like the working time consumption or limits on *unsocial working hours*. Other resources belong to *global rules* that involve all rosters. One example are *duty mix constraints* that limit the percentage of duties of a specified type, like weekend-duties. We will discuss some of these resources and their impact on the solvability of the TEP. Another important aspect will be the influence of input parameters on the size of D.

Furthermore, computational results from real-world instances complete our presentation. To solve the IP, we use the CPLEX solver by IBM. On a 8-core Intel Xeon workstation almost all instances occuring in daily operation can be solved either to optimality or with a small gap of at most 5%. One reason for the good performance is the quality of the lower bound produced by the linear relaxation. For most instances the gap between the initial lower bound and the optimal solution is distinctly lower than 10%. Since there are different control areas in Germany with different sizes and settings, some instances could be solved to optimality within 10 minutes while other require even more than two hours to find a feasible solution. We conclude that this approach is a successful example for the use of mathematical optimization techniques in real-world, since our method is implemented at the enforcement agency in Germany and part of the planning process for the enforcement.

References

1. Ralf Borndörfer and Guillaume Sagnol and Elmar Swarat, A Case Study on Optimizing Toll Enforcements on Motorways, 3rd Student Conference on Operational Research, OpenAccess Series in Informatics (OASIcs), 22, 1–10 (2012)
2. P. Cappanera and G. Gallo, A Multicommodity Flow Approach to the Crew Rostering Problem, Operations Research, 52 (4), 583–596 (2004)