10th International Conference of the Practice and Theory of Automated Timetabling
PATAT 2014, 26-29 August 2014, York, United Kingdom

Dantzig-Wolfe decomposition of Meeting planning problems

Niels-Christian Fink Bagger · **Matilda Camitz** ·
Thomas Stidsen

Abstract Planning meetings of groups of persons is an activity which secretaries all over the world performs every day. For the cases where persons should participate in several meetings, this can actually become a hard planning problem to solve. In this abstract we will briefly present a Branch & Price approach for a general setup. A complete article about this approach has been submitted [1].

Keywords Meeting planning, Branch & Price

1 Introduction

Meeting planning *can* be a non-trivial task, if enough people are involved and if several of the people have to participate in several meetings. In this case, the planning problem becomes NP-hard [1].

2 Dantzig-Wolfe decomposition

The overall idea in this abstract is to apply Dantzig-Wolfe decomposition and create a master optimization problem and a sub optimization problem, where the differences between the different versions of meeting planning problems are ”hidden” in the sub-problem.

The two optimization problems, for which we will apply Column Generation, inside a Branch & Bound algorihm (i.e. Branch & Price) are now:

- A master problem which plans the meetings according to person schedules
- A sub-problem, per person, which generates new person schedules for each person.

Unfortunately we are not able to present the sub-problem models in this abstract, since these models are rather large and we have limited space here.

2.1 Master Problem

The master problem assume to have, for each person, a number of meeting plans. Each meeting plan specify *when* a meeting with the person is possible, but not which meeting. Assume that there are a number of persons e who has to participate in some meetings g. The meetings can occur in a number of timeslots b but naturally each

Niels-Christian Fink Bagger, Camilla Camitz · Thomas Stidsen
Technical University of Denmark
E-mail: thst@dtu.dk

person can at most participate in *one* meeting in each timeslot. The binary variable $x_{g,b}$ defines if meeting g takes place in timeslot b and there is a gain $\alpha_{g,b} > 0$ by having meeting g in time slot b. Whether a person should participate in a meeting is defined by an incidence matrix A^e_g which is 1 if person e is part of the group g constituting the meeting. The The master problem will attempt to select the best meeting plans $\lambda^{e,p}$ for each person e and plan p. A meeting plan is defined by an incidence matrix $M^{e,p,b}$ which is 1 if person e in plan p can have a meeting in timeslot b. The cost of a meeting plan is $\beta^{e,p} < 0$. The full master problem is now:

$$\max \quad \sum_{g,b} \alpha_{g,b} \cdot x_{g,b} + \sum_{e,p \in P_e} \beta^{e,p} \cdot \lambda^{e,p} \tag{1}$$

$$\text{s.t.} \quad \sum_{b} x_{g,b} \leq 1 \quad \forall g \tag{2}$$

$$\sum_{g} A^e_g \cdot x_{g,b} - \sum_{p \in P_e} M^{e,p,b} \cdot \lambda^{e,p} \leq 0 \quad \forall e,b \tag{3}$$

$$\sum_{p \in P_e} \lambda^{e,p} \leq 1 \quad \forall e \tag{4}$$

$$x_{g,b}, \lambda^{e,p} \in \{0,1\} \tag{5}$$

2.2 Sub-problem

The sub-problem generates meeting plans for each person. For different types of meeting problems, different requirements can be put here. Due to space limitations, it is impossible to include the three different sub-problems of the test problems in this abstract and we refer to the full article [1].

3 Tests

We test the approach on three different problems:

- Parent-teacher meetings at high-schools
- Supervisor-student meetings at high-schools
- Exam planning at high-schools

The developed Branch & Price algorithm is compared to two alternative approaches: ALNS and MIP.

ALNS (Adaptive Large Neighborhood Search) is a relatively new meta-heuristic, see [2]. An ALNS can be considered to be a special type of hyper heuristic and ALNS algorithms has been very successful in the area of Vehicle Routing. An ALNS has been developed for the Parent-teacher meeting problem and the Supervisor-student meeting problem, see [3].

MIP (Mixed Integer Programming) is a direct model of the two different problems, which is solved directly in the Gurobi solver [4].

3.1 Parent-teacher meetings

In Danish high-schools (9'th to 12'th grade), the school will typically arrange 2 meetings pr. year between the student and parents, and the teachers which they wish to meet. The objective is to minimize the time for the schedules of the students. The overall results when testing on 100 real-world problems are shown below in Table 1. Unfortunately, because of space restrictions we are not able to describe the details for each of the 100 real-world problems. Statistically, the number of meetings varies between 23 and 1187, the number of timeslots varies between 8 and 51 and the number of persons varies between 21 and 307. The largest dataset contains 779 meetings spread over 51 timeslots for 291 persons.

Table 1 Summary of results for parent-teacher meetings. 'Best obj' denotes the amount of instances where the algorithm provided the best objective value (including draws). 'Best UB' denotes the amount of instances where the algorithm found the best upper bound (including draws). Columns 'Gap $\leq q$ shows the amount of instances for which the respective algorithm provided a gap $\leq q$. 'Avg. Gap to best UB' is found for each algorithm by finding the best available UB for each instance, calculating the gap to the solution provided, and averaging these gaps.

	Best obj	Best UB	Gap = 0%	Gap ≤ 2%	Gap ≤ 5%	Avg. Gap to best UB
ALNS	46	-	-	-	-	2.31%
MIP	21	19	17	23	35	9.37%
B&P	54	94	16	54	92	2.32%

3.2 Supervisor-student meetings

In the third year of high-school, the students have to write a bigger assignment, for which they will get two supervisors in different subjects. Again the job is to plan the meetings between the teachers and the students. The overall results after testing on 100 real-life datasets are shown in Table 2 below. Unfortunately, because of space restrictions we are not able to describe the details for each of the 100 real-world problems. Statistically, the number of meetings varies between 21 and 303, the number of timeslots varies between 8 and 102 and the number of persons varies between 29 and 367. The largest dataset contains 258 meetings spread over 74 timeslots for 288 persons.

Table 2 Summary of results for supervisor-student meetings. Columns are equivalent to those in Table 1.

	Best obj	Best UB	Gap = 0%	Gap ≤ 2%	Gap ≤ 5%	Avg. Gap to best UB
ALNS	37	-	-	-	-	1.26%
MIP	16	14	10	22	42	7.13%
B&P	68	95	23	80	97	1.15%

3.3 Exam planning

Exam planning is the problem of scheduling the exams such that no student will have to go to two exams on the same day and that as many students as possible will have good preparation time between the exams. Just solving the relaxed probelm using Column Generation is so slow that full problems cannot be solved. Hence we will not present any results here.

3.4 Conclusion

Many different time-table problems can be considerede to belong to the meeting planning problem category. The particular Dantzig-Wolfe decomposition approach works fine on two of the test problems, but not on the last problem. We conclude that the method works well, if there are not too many persons involved and if each person has a significant number of meetings, i.e. more than say 3.

References

1. Niels-Christian Fink Bagger et. al., ”A Mathematical Programming Approach to the Generalized Meeting Planning Problem”, submitted to Computers & OR
2. Stefan Ropke and Pisinger, David, ”An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows”, TRANSPORTATION SCIENCE, Vol. 40, No. 4, pp. 455-472, 2006
3. Simon Kristiansen, Matias Sorensen, Michael Herold and Thomas Stidsen, ”The Consultation Timetabling Problem at Danish High Schools”, Journal of Heuristics, Vol. 19, No. 3, pp. 465-495, 2013
4. Gurobi Optimization, Inc., ”Gurobi Optimizer Reference Manual”, 2012.