Airport Ground Staff-sizing with Hierarchical Skills Using Column Generation

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1 Introduction

We address the strategic problem of workforce planning for airport ground staff, which is part of a large business project aimed at developing an efficient and flexible ground handling resource management system for a major airline in China. The considered staff-sizing problem is an anonymous shift planning problem to satisfy a given demand profile with respect to a set of flexible shift rules, like [1].

To capture the high variations of demands at different times of the day, our weekly planning horizon is divided into equal time periods [2]. The demand for each period is given by an external demand modelling process [3]. Instead of covering all demands, we prefer a minimum workforce satisfying a target coverage rate, which is defined as the ratio of covered demands to aggregate demands for certain time periods, since part-time employees are not available for us in contrast to the case in most literature. We categorize our airport ground staff into hierarchical skill levels by productivity or seniority. Compared to [4] which also considered downgrading, our model can handle problems of more than two levels in a simpler and more efficient way.

2 Problem Formulation and Solution Approach

The proposed model follows the idea of many classical tour scheduling problems [5]. We solve the model using a column generation approach [6] in which a master problem and pricing problems are solved iteratively.

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2.1 Master Problem

Consider a weekly planning horizon composed of a set of time periods $T = \{1, \ldots, |T|\}$ whose length is typically 15 minutes. Employees and demands are categorized into several levels $L = \{1, \ldots, |L|\}$ with level 1 the highest. Denote J the set of feasible tours. For a tour $j \in J$, a_{tj} is 1 if $t \in T$ is a working period and 0 otherwise. The decision variable x_{lj} takes the value of the number of level $l \in L$ employees to be assigned to tour j. Supposing our target is the overall coverage rate r_l for the whole week for each level l, we could calculate the threshold value of aggregate understaffing demands as $\gamma_l = \lfloor (1 - r_l) \sum_{t \in T} d_{lt} \rfloor$, with d_{lt} the given level l demand at period t. Denoting u_{lt} the understaffing demand of level l at period t, $\sum_{t \in T} u_{lt}$ shall not be greater than the threshold γ_l. The model is then formulated as follows.

$$\min \quad \sum_{l \in L} w_l y_l + M \sum_{l \in L} \beta_l \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in J} a_{tj} x_{1j} + u_{1t} - o_{1t} = d_{1t} \quad \forall t \in T \tag{2}$$

$$\sum_{j \in J} a_{tj} x_{lj} + u_{lt} - o_{lt} + o_{l-1,t} = d_{lt} \quad \forall t \in T, l \in L \setminus \{1\} \tag{3}$$

$$\sum_{j \in J} x_{lj} - y_l = 0 \quad \forall l \in L \tag{4}$$

$$\sum_{t \in T} u_{lt} + \alpha_l - \beta_l = \gamma_l \quad \forall l \in L \tag{5}$$

$$u_{lt} \leq d_{lt} \quad \forall t \in T, l \in L \tag{6}$$

$$x_{lj}, y_l, u_{lt}, o_{lt}, \alpha_l, \beta_l \in \mathbb{N} \quad \forall l \in L, t \in T, j \in J \tag{7}$$

The objective (1) is to minimize the weighted sum of the numbers of employees y_l of level l defined in equation (4). Constraint (5) and objective (1) assure the coverage rate target, where α_l/β_l are the understaffing demands below/over the threshold, and M is a sufficiently large penalty. Constraints (2) and (3) state that employees of higher levels are permitted to cover demands of lower levels. It is worth emphasizing that u_{lt} and o_{lt} might both take positive values in some cases, i.e. o_{lt} should not be simply considered as overstaffing demands. Actually, o_{lt} is the number of employees of level l or higher to be assigned to demands of level $l+1$ or lower at period t.

A solution to this model does not explicitly give a specific assignment of employees to hierarchical demands. For each period t, we could specify an assignment in a greedy fashion iteratively, i.e. for demands and employees not specified yet at each iteration, assign a highest level demand to a highest level employee. The procedure is valid since constraints (2) and (3) assure that the highest K levels satisfy $\sum_{l=1}^{K} \sum_{j \in J} a_{tj} x_{lj} - \sum_{l=1}^{K} (d_{lt} - u_{lt}) = o_{Kt} \geq 0$.

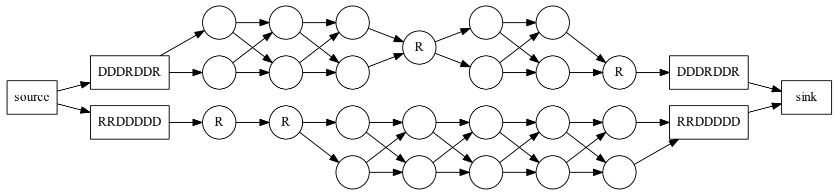

Fig. 1 Illustration of pricing graph

2.2 Pricing Problem

Denote the dual values of constraints (2) and (3) as π_{lt}, and that of (4) as ρ_l. The pricing problem for each level l finds tours with negative reduced cost $-\sum_t a_{tj}\pi_{lt} - \rho_l$ at each iteration of column generation.

We construct a directed acyclic graph for each level l in which the weight of a route from source to sink equals to the reduced cost of its corresponding tour, with respect to a set of shift rules: allowable shift types, minimum rest time, two days off a week, etc. The idea is illustrated in Figure 1. Each node represents an allowable shift in a single day with weight $-\sum_{t\in T_{shift}} \pi_{lt}$ where T_{shift} is its working periods. Two shift nodes are connected if there is long enough rest time in between. To cope with the day-off rule, we enumerate all possible shift sequences and construct a subgraph for each of them. A shift sequence, e.g. "DDDRDDR" in Figure 1, indicates on which days the employee have a shift ("D") or a day-off ("R"). All subgraphs are connected to the common source and sink nodes. We set the source node with weight $-\rho_l$, and all other nodes with 0. Then we solve the K shortest paths problem to price out a number of tours using a deletion path algorithm [7].

Last but not least, we proposed a branch-and-price scheme [6] by branching on any fractional y_l. If all y_l are integer, we keep branching on an y_l whose value domain is not a singleton. Note that branching on y_l does not affect the pricing problem except for the dual values. We stop branching if all y_l are fixed, then search for a feasible solution with the generated columns. We use the best-bound strategy to select the next node. As a result, each leaf node on the branch-and-bound tree corresponds to a workforce mix, and the lower bounds of the leaves automatically guide our algorithm to the most promising ones. Furthermore, for instances with more than 3 levels, we decompose the problem into several K-level (typically 2 or 3) subproblems in a "rolling window" fashion in order to keep the computation time manageable.

3 Experimental Results

All of our experiments are based on real-world demand data at an airport in China. The algorithm is coded in C with the commercial solver Xpress. We

Table 1 Summary of experiment results

	#Levels	#Peaks	UB	LB	Gap (%)	Time (sec.)
Scenario 1	3	7	320.0	319	0.31	288.2
Scenario 2	4	28	917.6	909	0.94	936.1
Scenario 3	4	7	1307.8	1306	0.14	434.9
Scenario 4	8	7	50372.2	50293	0.16	214.6

Table 2 Workforce mix with different shift rules

Downgrade	#Overtime	Peak/Coverage(%)	Workforce Mix
Y	1	70/80	9/59/37
N	1	70/80	9/62/38
Y	0	70/80	9/66/41
N	0	70/80	9/69/41
Y	1	80/90	11/69/44
N	1	80/90	11/73/44
Y	0	80/90	11/78/49
N	0	80/90	11/82/50

perform experiments for 4 different scenarios, 5 runs for each, on a 3.5 GHz 12 threads machine with 32 GB of RAM. For all scenarios, we require a weekly overall coverage, and a smaller target for each defined peak interval. Table 1 shows the number of skill levels, the number of peaks in a week, and the average upper bound, lower bound, gap, and running time of the experiments. Table 2 compares the workforce mix generated for the same demand profile with 3 levels under different shift rules (whether downgrading is permitted; the number of overtime shifts allowed; target coverage for each peak interval and the whole week), which is the major concern of our system user.

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