

---

## Online Problems in Timetabling: Bus Priority at Signalised Junctions

Randall Cogill, Jakub Mareček,  
Martin Mevissen, and Hana Rudová

**Abstract** We argue for the study of on-line problems in timetabling and give some novel examples thereof. The problem of prioritising delayed buses at signalised junctions underlies a timetabling competition held in conjunction with the International Conference on the Practice and Theory of Automated Timetabling.

### 1 Introduction

A number of communities study algorithms under a variety of assumptions. A setting traditional in Theoretical Computer Science involves “given the complete input, we produce the complete output”, with the objective and its optimality verifiable from the complete output alone. These are known as off-line problems. Traditional assumptions in Automatic Control include “a controller continuously adjusts a control variable, based on continuously available measurements of a process variable,” while an objective related to the behaviour of the controller evaluates performance over the long run. Algorithms that process the input in a serial fashion, as it becomes available, are sometimes known as online algorithms. Algorithms with this characteristic are also sometimes referred to as stream processing. As the control variable affects what input becomes available in the future, the simple certificates of optimality required within the class of NP are not available and one often studies the stability or regret of algorithms instead.

---

R. Cogill, J. Mareček, and M. Mevissen,  
IBM Research – Ireland,  
B3 IBM Campus Damastown, Dublin 15, Ireland  
Tel.: +353 85 1593 000  
E-mail: jakub.marecek@ie.ibm.com

H. Rudová  
Faculty of Informatics, Masaryk University,  
Botanická 68a, 602 00 Brno, Czech Republic

Traditionally, much research in timetabling has focussed on off-line problems. In educational timetabling, a timetable is produced, once student registrations become known, and stay fixed until the end of the term. In public transportation, a timetable for the public transportation system is produced with some variation over the day of the week and hour of the day, once demands for transportation, vehicle and personnel availability are estimated. Only limited attention is being given to the questions of robustness and controllability.

In many situations, however, it is impossible to adhere to the timetable, as planned. In educational timetabling, a teacher or a room may become unavailable. In public transportation, congestion may arise and increase the travel times sharply. In many situations, it is possible to resort to a “recourse action”, recovering the feasibility of the timetable, perhaps at an additional cost. In educational timetabling, a course may be cancelled or given at another time. In public transportation, wait-time of a delayed bus at a signalised junction may be reduced by changing the phase of the traffic lights. While in interactive problems [38,30], such a recourse action is implemented by a human operator, we argue for the use of automated control. When one considers one revelation of uncertainty and one recourse action, the control problem has been studied under the name of the minimum perturbation problem [28,25]. It does not seem to be sufficient to evaluate a policy based on its performance with respect to a single recourse action, though. We hence argue for the study of on-line problems in timetabling, where the timetable is updated repeatedly following the repeated revelation of uncertainty, where a finite number of such recourse actions are evaluated over a finite horizon, while considering the distance from the previously announced timetable.

## 2 A Toy Example

Let us introduce the toy example of a morning routine of a couple, where both characters need to wake up, use the bath room, prepare a breakfast, eat the breakfast, and travel to their respective offices for 9 a.m. The deterministic, off-line version of the problem may comprise of:

- It takes 2.2 minutes to wake up, after the alarm clock rings.
- Each person occupies the bath room for 10 minutes.
- The preparation of a breakfast takes 7.5 minutes.
- The eating of a breakfast takes 5 minutes.
- Getting to work takes 48 and 18 minutes for the two characters, respectively.
- At most one person can use bath rooms at one time, and at most one person can cook porridge at one time, due to incompatible preparation preferences, and at most one person can drive the car to work.
- Minimise the time between the alarm clock ringing and the arrival to work in minutes plus penalties for arriving late to work (10 per minute).

Such a problem is clearly in NP, i.e., one can verify that the optimal solution consists of the first characters waking up at 7:47.3 and leaving home at 8:12, with the other characters waking up at 8:17.3 and leaving home at 8:42. Such a precise statement may already raise eyebrows.

Let us consider a more elaborate setting, where the couple have collected data about the duration of their activities using the recently fashionable activity trackers over the course of the past week, e.g.,

- the waking up has taken 0, 1, 0, 10, 0, 0, 1, 0, 10, and 0 minutes after the alarm rang, in the past week
- the bath room is occupied for 10 minutes, in all 10 measurements
- there are two alternatives for the preparation and eating of a breakfast: either cooking a porridge (10, 15, 10, 8, 12, 10 minutes), or slicing bread (1, 2, 2, 5 minutes)
- the eating of a breakfast takes 5 minutes, in all 10 measurements
- there are four alternatives for the journey to work for the first character: either cycling (50, 40, 90), driving (30), or taking the public transport (40), or taking a taxi (20)
- there are four alternatives for the journey to work for the second character: either cycling (20, 15), taking the public transport (20, 25), or driving (15), or taking a taxi (10).

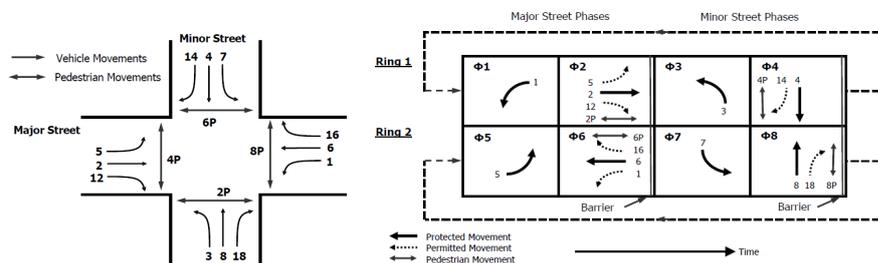
The objective is to minimise the sum of

- time between waking up and arrival to work
- penalties for arriving late to work (10 per minute)
- the spending on public transport (1 per use), own car (5 per use), and taxis (10 per use).

Such a problem can be in NP, when one assumes that the 10 empirically observed realisations of the multivariate uncertainty (e.g. 0 for waking up, followed by 10 for the bath room, 10 for the porridge, 5 for the eating, and 50 for the cycling) are equiprobable and that no others are possible, as one often does in stochastic programming. That may improve upon the situation with taking the expectations for the off-line problem, but still seems rather questionable an assumption.

Let us consider a yet more elaborate setting, where the couple uses the history to develop policies, where based on the sum of the time it took to wake up and the time it took to prepare the breakfast, one decides on the mode of transport. The problem is further complicated if we assume that the multi-variate random variables are multi-variate Gaussian, of which we have sampled 10 values. Now we can no longer assume that the problem is in NP, because we cannot guarantee that the solution is optimal. Indeed, it is not even clear, whether the problem is decidable, e.g., whether one can encode a feasible solution in a finite amount of space. It is, nevertheless, much closer to the morning routine and the decision making therein.

Finally, one may consider the notion of a state. One morning, the first character may have been in a panic mode: This individual has woken up late



**Fig. 1** An example set of movements (left) and phases (right) at a 4-way junction. Cited in verbatim from Exhibits 31-1 and 31-2 of Volume 4 of Highway Capacity Manual [32].

(10 minutes), chose to have bread (1 minute preparation time), and then took a taxi to work (20 minutes). Another morning, the first character may have been relaxed, after he has woken up on time (7:47.3). Perhaps, based on the realisations of the uncertainty in the time it took to wake up, one decides whether to cook a porridge or slice bread? Based on the realisations of the uncertainty in the time it took to wake up and to prepare the breakfast, one decides on the mode of transport? Perhaps, the difficulties in waking up and cooking among the two characters may be related, e.g., the morning after a party attended by both characters? Such details affect the state space, and hence the computational complexity. For finite state spaces, however, it maybe possible to show the problems are in P-SPACE.

## 2.1 Model

Let us formalise the toy problem, while breaking the input to three parts: the static part, the historical data, and the streamed data updated at each time step.

## 2.2 The Input

The static part of the input comprises the description of the activities, constraints on activities, and costs:

- $\mathcal{A}$ , a set of activities to perform, e.g., the preparation of breakfast, the use of bathroom
- $\mathcal{E} \subseteq \mathcal{A}$ , a subset of activities at most one person can perform at the same time, e.g., the use of bathroom
- $\mathcal{O}(a)$ , the set activities, which become available for processing after finishing activity  $a \in \mathcal{A}$ , e.g., after waking up, the use of bathroom and the preparation of a breakfast
- $\mathcal{P}_c^a$  the description of the possible variants character  $c \in \mathcal{C}$  can choose from in activity  $a \in \mathcal{A}$ , e.g., the mode of transport

- $C_c^p(t)$  the cost of the possible variant  $p \in \mathcal{P}_c^a$  for the activity  $a \in \mathcal{A}$  taken at time  $t \in \mathcal{T}$  by character  $c \in \mathcal{C}$ .

For each day, i.e., for each finite planning horizon, there is the timetable available. Note that the timetable is considered part of the input, since we are considering recourse actions taken when deviations from the timetable are experienced.

- $\mathcal{C}$ , the set of persons the timetable is for
- $\mathcal{T}$ , the set of time-steps within the day
- $T_c(t) \in \mathcal{A}$ , the activity character  $c \in \mathcal{C}$  should perform at time  $t \in \mathcal{T}$ , according to the timetable.

At each time-step  $t \in \mathcal{T}$ , the following historical data is also available:

- $A_c(t) \in \mathcal{A}$ , the actual activity character  $c \in \mathcal{C}$  performed at time  $t$  in a past realisation of the execution of the timetable.

The historical data may include several years worth of per-day and per-time-step data, input and output, at circa second frequency, with periods of no data available at night and during system outages.

### 2.3 The Output

At each time-step  $t \in \mathcal{T}$ , the following should be made available for each character and the activity it currently performs:

- $p_c^a(t) \in \mathcal{P}_c^a$ , the possible variant of activity  $a$  chosen by character  $c \in \mathcal{C}$ .

### 2.4 The Evaluation

Assuming the values of all random variables are known, each policy is evaluated with respect to the costs incurred by the actions:

$$\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} C_c^{p_c^a(t)}(t). \quad (1)$$

Clearly, the smaller, the better.

### 2.5 Related Work

There is a rich history of work on automatic control and optimisation under uncertainty, and their complexity, albeit not applied to timetabling, or the particular morning routine problem. Problems related to the above, with finite state-space and finite horizon, have been shown to be in PSPACE [2]. We point to: [5] for on-line learning and games, [3] for model predictive control, and [2] for complexity theory.

### 3 The Competition Problem

We also present a problem encountered in operations control in public transportation, with non-trivial constraints on the actions to be performed. In many cities, the operator of the public transportation system and the transport authority work together to improve the performance of the transportation system. Considering most large cities have remotely controllable traffic lights and delayed buses, with the increasing number of cities having real-time bus positions available, this increasingly involves adjusting the traffic lights such that the waiting times of delayed buses are reduced.

#### 3.1 Model

As usual in the literature, e.g. [39], we discretise time uniformly during the hours of operations of the buses. The time-tabled  $T_b(t)$  and actual  $B_b(t)$  positions of each bus  $b$  are available at each time  $t$ . The road network is modelled in two ways:

- as a “primal” directed graph with nodes  $(N_a)_{a \in \mathcal{N}}$  and arcs  $(L_j)_{j \in \mathcal{L}}$
- as a “dual” directed graph where junctions  $(J_i)_{i \in \mathcal{J}}$  partition arcs  $\mathcal{L}$  of the primal directed graph and  $\mathcal{I}(J)$  and  $\mathcal{O}(J)$  match the connectivity of the partitions within the primal directed graph.

Along each arc, there are  $W_a \in \mathcal{I}(t)$  vehicles queuing at time  $t$ , out of which a number  $X_a \in \mathcal{I}(t)$  are considered priority vehicles, such as delayed buses. We know the number of vehicles queuing at each junction, but we know the individual coordinates and destinations only for the priority vehicles, while the route choices of the remaining vehicles are described by the ratio  $r_{ab}$  of vehicles at node  $N_a$  wishing to go in direction  $b$ .

This operates as network of queues. At every junction  $J$ , there is a queue for each pair in  $\mathcal{I}(J) \times \mathcal{O}(J)$ . Vehicles move based on the setting of the phase  $p_i$  at each  $J_i$ ,  $p = (p_i)_{i \in \mathcal{J}} \in \mathcal{P}$ . Queues are assumed to have infinite capacities and movements  $\mu_{ab}(t)$  are assumed to take one out of two possible values, 0 and the saturation rate  $s_{ab}$ . At each time  $t$ , for each queue  $(a, b) \in \mathcal{I}(J) \times \mathcal{O}(J)$ , there are  $\mu_{ab}(p) \in \{0, s_{ab}\}$  vehicles moved from  $a$  to  $b$ . See Figure 1 for an additional illustration.

Our horizon is finite  $1 \dots T$ , i.e. one day, and our goal is to minimise a convex combination of the sum of the priority vehicles queued over time and the sum of all vehicles queued over time and certain other performance indicators.

#### 3.2 The Input

As with the first problem, the input has three parts: the static part, the historical data, and the streamed data updated at each time step.

### 3.2.1 The Static Information

The static part of the input comprises the description of the road network:

- $\mathcal{N}$ , the set of nodes in the primal representation of the network
- $\mathcal{L}$ , the set of links in the primal representation of the network
- $\mathcal{J}$ , the set of junctions in the dual representation of the network
- $\mathcal{I}(J_i)$ , the adjacent junctions, inbound, at junction  $J_i \in \mathcal{J}$  in the dual representation of the network
- $\mathcal{O}(J_i)$ , the adjacent junctions, outbound, at junction  $J_i \in \mathcal{J}$  in the dual representation of the network
- $\mathcal{P}_a$  the set of phases available at node  $N_a$ , for all  $N_a \in \mathcal{N}$ .

### 3.2.2 The Per-Day Information

For each day, i.e., for each finite planning horizon, there is the timetable available:

- $\mathcal{R}$ , the set of routes operated that day
- $\mathcal{V}$ , the set of vehicles (buses)
- $\mathcal{L}_r(t)$ , the subset of vehicles operated on route  $r \in \mathcal{R}$  at time  $t \in \mathcal{T}$
- $\mathcal{T}$ , the set of time-steps within the day
- $T_v(t)$ , the  $(x, y)$  coordinates of vehicle  $v \in \mathcal{V}$  timetabled for time  $t \in \mathcal{T}$ , with the vector  $T(t) = (T_v(t)) \in \mathbb{R}^{|\mathcal{V}| \times 2}$
- $D_v, a \in \mathcal{I}$ , the set of timetabled departures of vehicles  $v \in \mathcal{V}$  at node  $N_a$  at time  $t$ . The cardinality of the set varies

### 3.2.3 The Per-Period Information

At each time-step, the following is available:

- $B_b(t)$ , the  $(x, y)$  actual coordinates of bus  $b \in \mathcal{V}$  at time  $t \in \mathcal{T}$ , with the vector  $B(t) = (B_b(t)) \in \mathbb{R}^{|\mathcal{V}| \times 2}$
- $S_b(t)$ , the state of bus  $b \in \mathcal{V}$  at time  $t \in \mathcal{T}$ , with  $S_b(t) \in \{1, 2, 3, 4\}$ , which represents running, stopped with door closed, stopped with door open, out of service, respectively
- $Q_{ab}$ , the queues at node  $N_a$  in direction of  $b \in \mathcal{O}(J_i)$
- $W_a \in \mathcal{I}(t)$ , the number of vehicles at node  $N_a$  at time  $t \in \mathcal{T}$ , which includes both exogenous and endogenous arrivals, and both standard and priority vehicles
- $r_{ab}$  with  $\sum_b r_{ab} \leq 1$  being the ratio of vehicles at node  $N_a$  wishing to go in direction  $b$
- $A_a(t)$ , the number of vehicles entering the transportation network at node  $N_a$  during slot  $t \in \mathcal{T}$  (“exogenous arrivals”)
- $1 - \sum_b r_{ab}$ , the proportion of vehicles exiting the transportation network at  $N_a$  (“exits”)
- $X_a \in \mathcal{I}(t)$ , the number of priority vehicles at node  $N_a$  at time  $t \in \mathcal{T}$ , such as delayed buses

- $s_{ab}$  with  $\sum_b s_{ab} \leq 1$  being the ratio of priority vehicles at node  $N_a$  wishing to go in direction  $b$

These allow for the computation of the service matrix  $\mu(p(t))$  at time  $t \in \mathcal{T}$ .

The historical data include several years worth of per-day and per-time-step data, input and output, at circa second frequency, with periods of no data available at night and during system outages.

### 3.3 The Output

At each time-step, the following should be made available:

- $p_a(t)$ , the phase activated at node  $N_a$  at time  $t \in \mathcal{T}$ , with the vector  $p(t) = (p_a(t))$

### 3.4 The Evaluation

Considering coefficients  $\delta, \eta, \omega, \phi \in \mathbb{R}$ , and assuming the values of all random variables are known, the policy is evaluated with respect to:

$$\delta\Delta + \eta\mathbf{H} + \omega\Omega + \phi\Phi \quad (2)$$

where:

- the sum of deviations from the timetable:

$$\Delta := \sum_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}} \|B_v(t) - T_v(t)\|^2 \quad (3)$$

- the sum of the head-ways:

$$\mathbf{H} := \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{L}_r(t)} \min_{w \in \mathcal{L}_r(t) \setminus \{v\}} \|B_v(t) - B_w(t)\|^2 \quad (4)$$

- the sum of the numbers of vehicles waiting at signalised junctions over time:

$$\Omega := \sum_{t \in \mathcal{T}} \sum_{J_i} \sum_{a \in \mathcal{I}} W_a \quad (5)$$

- the sum of the numbers of priority vehicles waiting at signalled junctions over time:

$$\Phi := \sum_{t=1}^T \sum_{J_i} \sum_{a \in \mathcal{I}} X_a. \quad (6)$$

Notice that by varying the weights  $(\delta, \eta, \omega, \phi)$ , one can model, e.g., the objective of the bus operator  $(1, -0.01, 0, 0)$  and the objective of the traffic management team  $(0, 0, 0.01, 1)$ . Clearly, for non-negative weights, the smaller, the better.

### 3.5 The Competition

Based on the problem presented in Section 3, a competition is being organised in conjunction with the International Conference on the Practice and Theory of Automated Timetabling. The competition is based on a web service. Upon registration, all participants in the competition are provided with the details described in Section 3.2 in JSON format, and can submit any phase-change decisions, as described in Section 3.3, in the JSON format at any time. The web service continuously evaluates the participants' actions using a microsimulation engine and records any details. After an initial trial period, the results of all participants, who do not choose to withdraw from the competition, will be posted in a leaderboard, publicly available both via the web service and the associated website. Notice, however, that the leaderboard is only indicative for much of the time; the final ranking will be performed based on the performance in the final weeks of the competition. We hope this set-up will provide an appropriate trade-off between the protection of the intellectual property rights of the participants, fairness, and transparency in the evaluation.

In addition to the web service, we also provide some auxiliary code for the convenience of the participants. For illustration and comparison purposes, we provide examples of code in Python for three simple benchmarks:

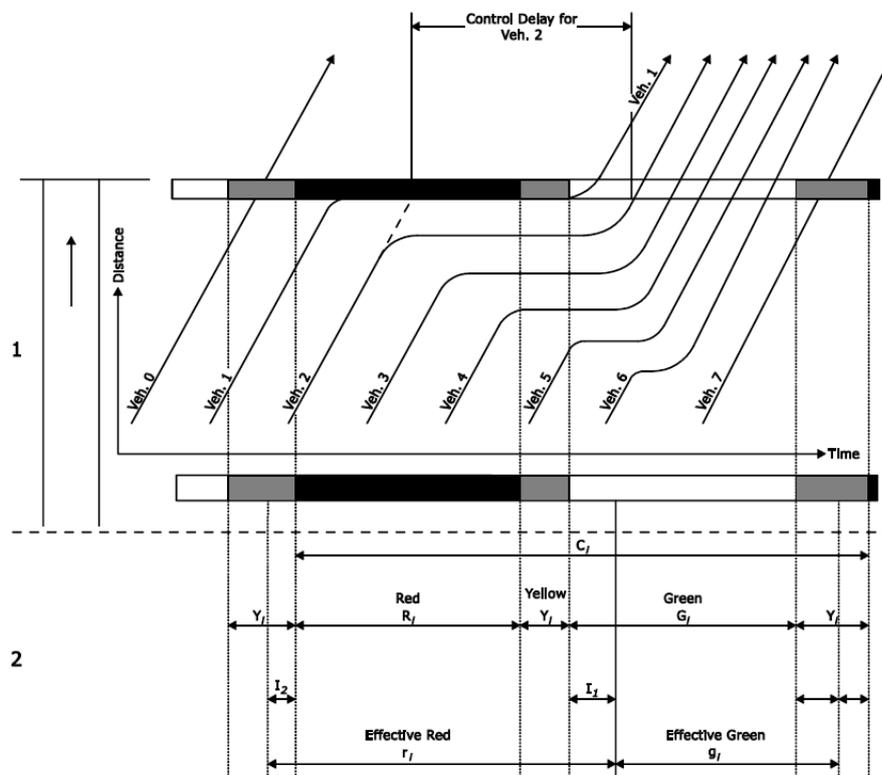
- Lazy: no phase-change decisions are ever submitted, i.e., no priority is given to any delayed buses and the default adaptive control mechanism is used.
- Fixed: phase-change decisions are submitted so as to implement regular timings.
- Actuated: phase-change decision is submitted at every point when a delayed bus is detected at an approach. Notice that this may lead to rapid phase changes, which may be undesirable.

These illustrate the mechanics of connecting to the web service. We note that one should like to improve over such example code considerably in order to participate in the competition successfully.

To aid the analysis, we also provide code, which produces and updates in real-time a variety of visualisations. The so called space-time and phase-split diagrams, exemplified in Figures 2 and 3, are perhaps the most widely recognised visualisations in transport engineering. We refer to the Highway Capacity Manual [32] for extensive guidance as to the use of the visualisations in the analysis.

### 3.6 Related Work

Although there is a long history of research into the problems of setting the timetable for buses and, independently, controlling traffic lights, we are not aware of any work presenting adjustments to the phases of traffic lights as a recourse in a stochastic bus timetabling. Giving priority to buses at signalised intersections, on its own, also has a long history of experiments [37, 12] and has

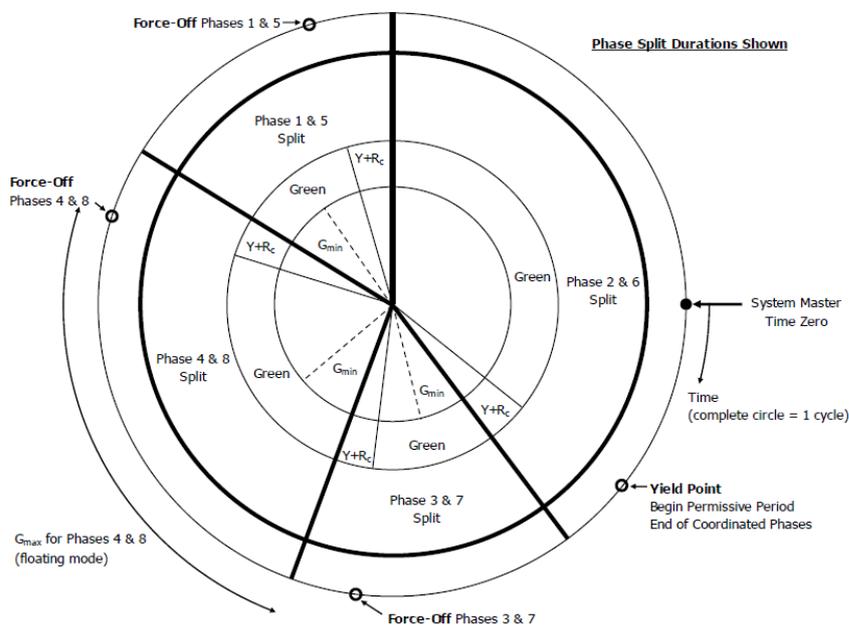


**Fig. 2** An example of a space-time diagram, where each line corresponds to one vehicle's trajectory (top). Times, when vehicles are waiting, correspond to horizontal lines, while the slopes of the non-horizontal lines correspond to the speed of the vehicles. For clarity, the corresponding phases are displayed below. Cited in verbatim from Exhibits 31-5 of Volume 4 of Highway Capacity Manual [32].

attracted much interest [34,36,35,16,1,9,40,26] recently. The combination of delaying the buses and giving priority to buses at intersections are two natural recourse actions in stochastic bus timetabling.

In the “off-line” bus timetabling, one distinguishes a number of problems, depending on:

- whether the service is schedule-based (with departure times set) or frequency-based (with only the headway or frequency set)
- the level of integration: in principle, one can adjust the fleet size, the lines to run (“Transit Network Design”), how often to run them (“Frequencies Setting Problem”), and in case of timed services, when exactly to run each service (“Transit Network Scheduling Problem”)
- the level of detail in modelling customer choice: is the demand for public transport from  $a$  to  $b$  elastic, i.e. can users choose a different mode of transport or decide not to travel, is the choice of a connection between  $a$  to  $b$  fixed, i.e. can users pick which connection to use based on the timetable?



**Fig. 3** An example of the phase-split diagram, where the circle corresponds to the cycle time and the arc segments correspond to phases. Cited in verbatim from Exhibits 31-9 of Volume 4 of Highway Capacity Manual [32].

Even with a simple bi-level model (e.g. non-elastic demand, but customers picking the fastest connection) and a subset of the integrated problem (e.g. Transit Network Design or Transit Network Scheduling Problem), the current methods do not make it to guarantee optimality, even in the steady-state [13]. There are hence a number of solutions to partial problems, e.g. without customer choice [6], solutions employing various decompositions [11], and heuristics [29]. There are many commercial products, including Giro Hastus, IVU Microbus, Lumiplan Heures, and PTV Visum. See [7,15] for surveys.

The research on the control traffic lights has evolved from pre-determined sequences with fixed timing, to pre-determined sequences timed in response to sensor data, to adaptive control, which aims to deal with the so called “oversaturated” conditions, where no approach clears within some time limit. The off-line problem of determining the fixed timing has been studied in operations research and transportation science since 1960s [23,14,4]. On-line formulations studied empirically in the 1980s, are still being used today in the form of Scats [21], Scoot [18], Prodyn [17], Rhodes [24], and similar systems. Recently, both simple [31,39,33,19,20] and not-so-simple [8] on-line algorithms with non-trivial theoretical bounds on their performance have been proposed. Although these are not yet widely used in practice, they have a number of advantages: they require only local information, which are readily available,

and have a per-iteration run-time independent on the size of the network. See [10,27] for surveys.

#### 4 Conclusions

We argue for the study of on-line problems in timetabling. Following the success of the previous timetabling competitions [22] affiliated with the International Conference on the Practice and Theory of Automated Timetabling, we hope that these problems will be of interest to the community, providing a fertile ground for theoretical developments, while having a considerable impact in practice.

*Acknowledgement* This work received funding from the European Union Horizon 2020 Programme (Horizon2020/2014-2020), under grant agreement number 688380.

#### References

1. Ahmed, F., Hawas, Y.: An integrated real-time traffic signal system for transit signal priority, incident detection and congestion management. *Transportation Research Part C: Emerging Technologies* **60**, 52 – 76 (2015)
2. Blondel, V.D., Tsitsiklis, J.N.: A survey of computational complexity results in systems and control. *Automatica* **36**(9), 1249–1274 (2000)
3. Camacho, E.F., Alba, C.B.: *Model predictive control*. Springer Science & Business Media (2013)
4. Cascetta, E., Gallo, M., Montella, B.: Models and algorithms for the optimization of signal settings on urban networks with stochastic assignment models. *Annals of Operations Research* **144**(1), 301–328 (2006)
5. Cesa-Bianchi, N., Lugosi, G.: *Prediction, Learning, and Games*. Cambridge University Press, New York, NY, USA (2006)
6. Chowdhury, S.M., I-Jy Chien, S.: Intermodal transit system coordination. *Transportation Planning and Technology* **25**(4), 257–287 (2002)
7. Chua, T.A.: The planning of urban bus routes and frequencies: A survey. *Transportation* **12**(2), 147–172 (1984)
8. Coogan, S., Gomes, G., Kim, E.S., Arcak, M., Varaiya, P.: Offset optimization for a network of signalized intersections via semidefinite relaxation. In: 2015 54th IEEE Conference on Decision and Control (CDC), pp. 2187–2192. IEEE (2015)
9. Dai, G., Wang, H., Wang, W.: A bandwidth approach to arterial signal optimisation with bus priority. *Transportmetrica A: Transport Science* **11**(7), 579–602 (2015). DOI 10.1080/23249935.2015.1049675
10. Diakaki, C., Papageorgiou, M., Aboudolas, K.: A multivariable regulator approach to traffic-responsive network-wide signal control. *Control Engineering Practice* **10**(2), 183–195 (2002)
11. Friedrich, M., Hofsaess, I., Webeck, S.: Timetable-based transit assignment using branch and bound techniques. *Transportation Research Record: Journal of the Transportation Research Board* (1752), 100–107 (2001)
12. Furth, P.G., Muller, T.H.J.: Conditional bus priority at signalized intersections: Better service with less traffic disruption. *Transportation Research Record: Journal of the Transportation Research Board* **1731**, 23–30 (2000)
13. Gao, Z., Sun, H., Shan, L.L.: A continuous equilibrium network design model and algorithm for transit systems. *Transportation Research Part B: Methodological* **38**(3), 235–250 (2004)

14. Gartner, N.H., Little, J.D.C., Gabbay, H.: Optimization of traffic signal settings by mixed-integer linear programming: Part i: The network coordination problem. *Transportation Science* **9**(4), 321–343 (1975)
15. Guihaire, V., Hao, J.K.: Transit network design and scheduling: A global review. *Transportation Research Part A: Policy and Practice* **42**(10), 1251–1273 (2008)
16. Guler, S.I., Menendez, M.: Pre-signals for bus priority: basic guidelines for implementation. *Public Transport* **7**(3), 339–354 (2015)
17. Henry, J.J., Farges, J.L., Tuffal, J.: The prodyn real time traffic algorithm. In: *Proceedings of the 4th IFAC/IFORS Conference on Control in Transportation Systems* (1984)
18. Hunt, P., Robertson, D., Bretherton, R., Royle, M.C.: The scoot on-line traffic signal optimisation technique. *Traffic Engineering & Control* **23**(4) (1982)
19. Kouvelas, A., Lioris, J., Fayazi, S., Varaiya, P.: Maximum pressure controller for stabilizing queues in signalized arterial networks. *Transportation Research Record: Journal of the Transportation Research Board* (2421), 133–141 (2014)
20. Lee, S., Wong, S., Varaiya, P.: Stage-based max-pressure adaptive traffic control policy at isolated signalized junctions. *Annual Meeting of Transportation Research Board (TRB)* (2016)
21. Lowrie, P.: Scats, sydney co-ordinated adaptive traffic system: A traffic responsive method of controlling urban traffic (1990)
22. McCollum, B., Schaerf, A., Paechter, B., McMullan, P., Lewis, R., Parkes, A.J., Gaspero, L.D., Qu, R., Burke, E.K.: Setting the research agenda in automated timetabling: The second international timetabling competition. *INFORMS Journal on Computing* **22**(1), 120–130 (2010)
23. Miller, A.J.: Settings for fixed-cycle traffic signals. *Operational Research Quarterly* pp. 373–386 (1963)
24. Mirchandani, P., Head, L.: A real-time traffic signal control system: architecture, algorithms, and analysis. *Transportation Research Part C: Emerging Technologies* **9**(6), 415–432 (2001)
25. Müller, T., Rudová, H., Barták, R.: Minimal perturbation problem in course timetabling. In: *Practice and Theory of Automated Timetabling V*, pp. 126–146. Springer (2004)
26. Oshyani, M.F., Cats, O.: Evaluating the performance and benefits of bus priority, operation and control measures. In: *The 95th Annual Meeting of the Transportation Research Board*. Transportation Research Board, Washington, DC (2016)
27. Papageorgiou, M., Diakaki, C., Dinopoulou, V., Kotsialos, A., Wang, Y.: Review of road traffic control strategies. *Proceedings of the IEEE* **91**(12), 2043–2067 (2003)
28. Phillips, A.E., Walker, C.G., Ehrgott, M., Ryan, D.M.: Integer programming for minimal perturbation problems in university course timetabling. *Annals of Operations Research* pp. 1–22 (2014)
29. Rapp, M.H., Gehner, C.D.: Transfer optimization in an interactive graphic system for transit planning. *Transportation Research Record* **619**, 27–33 (1967)
30. Rudová, H., Müller, T., Murray, K.: Complex university course timetabling. *Journal of Scheduling* **14**(2), 187–207 (2011)
31. Tassiulas, L., Ephremides, A.: Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks. *Automatic Control, IEEE Transactions on* **37**(12), 1936–1948 (1992)
32. Transportation Research Board: Chapter 31, Signalized Intersections: Supplemental, chap. 34, pp. 1–125. Transportation Research Board, Washington, DC (2010). DOI 10.5555/9780309160773.34
33. Varaiya, P.: Max pressure control of a network of signalized intersections. *Transportation Research Part C: Emerging Technologies* **36**, 177–195 (2013)
34. Wahlstedt, J.: Impacts of bus priority in coordinated traffic signals. In: *The 6th International Symposium on Highway Capacity and Quality of Service*, no. 16 in *Procedia Social and Behavioral Sciences*, pp. 578–587 (2011)
35. Wahlstedt, J.: Evaluation of different bus priority functions in coordinated traffic signals. In: *The 92nd Annual Meeting of the Transportation Research Board*. Transportation Research Board, Washington, DC (2013)

36. Wahlstedt, J.: Evaluation of the two self-optimising traffic signal systems utopia/spot and inflow, and comparison with existing signal control in stockholm, sweden. In: 16th International IEEE Conference on Intelligent Transportation Systems, pp. 1541–1546 (2013)
37. Wattleworth, J.A., Courage, K.G., Wallace, C.E.: Evaluation of bus priority strategies on Northwest Seventh Avenue in Miami (abridgment). *Transportation Research Record: Journal of the Transportation Research Board* **626**, 32–35 (1977)
38. White, G.M., Wong, S.K.: Interactive timetabling in universities. *Computers & Education* **12**(4), 521–529 (1988)
39. Wongpiromsarn, T., Uthaicharoenpong, T., Wang, Y., Frazzoli, E., Wang, D.: Distributed traffic signal control for maximum network throughput. In: *Intelligent Transportation Systems (ITSC), 2012 15th International IEEE Conference on*, pp. 588–595. IEEE (2012)
40. Xu, M., Ye, Z., Sun, H., Wang, W.: Optimization model for transit signal priority under conflicting priority requests. In: *The 95th Annual Meeting of the Transportation Research Board. Transportation Research Board, Washington, DC* (2016)