Probabilistic Curriculum-based Examination Timetabling

Bernd Bassimir · **Rolf Wanka**

Abstract In the literature, the examination timetabling problem (ETTP) is mostly described as a post enrollment problem (PE-ETTP). As such, it is known at optimization time how many students will take an exam and consequently how big a room is needed and which exams should not be held at the same time because of overlapping student lists. To compute a timetable using this approach, students need to register for exams before the timetable is generated. A direct consequence is that at registration time students have no idea when their exams are being held. Furthermore as timetables are often released at the end of the semester, it is hard for lecturers to plan their other responsibilities accordingly. This leads to a negative reaction from both the student body and the staff holding the exams. In this paper, we describe a curriculum-based examination timetabling variant that is similar to the curriculum-based examination timetabling problem model (CB-ETTP) introduced by Cataldo et al. [1]. The aim of the model introduced in this work is to combine the positive aspects of PE-ETTP and CB-ETTP by the use of machine learning while reducing the problems of the CB-ETTP, namely the overestimation in the number of students taking an exam. We describe an approach to calculate the number of students taking an exam by using old planning data. Furthermore we give an example for integrating the knowledge from past experience as a new soft constraint. Through the addition of this new soft constraint, we get a measure for the robustness of the timetable in respect to the uncertainty in the data. Finally, we present experiments based on real world data from the University of Erlangen-Nuremberg (FAU) showing that the approach gives a good estimation for the number of students with only slight deviations from the actual numbers.

Keywords Examination Timetabling, Curriculum-based Timetabling, Machine Learning

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Bernd Bassimir (✉), Rolf Wanka
Department of Computer Science, University of Erlangen-Nueremberg, Germany
E-mail: bernd.bassimir@fau.de · rolf.wanka@fau.de

1 Introduction

In every academic term, universities are faced with a number of different aspects of academic timetabling. Academic timetabling is divided into two distinct, but similar problems, namely the *Course Timetabling Problem* (CTTP) (for a comprehensive overview, see Mühlenthaler's monography [7]) at the beginning and the *Examination Timetabling Problem* (ETTP) at the end of the term.

CTTP is the task to assign a start time and one room to each lecture that is held in this term, creating a timetable that can be repeated each week over the length of the term. Besides the problem of checking whether there is an admissible timetable at all, there is also an optimization variant, if there is a way to assess timetables and look for "good" (or best possible) timetables.

In the ETTP, a start time and one or more rooms must be assigned to each exam. An exam is held only once per term and as such the timetable, in contrast to the CTTP, needs not to be repeatable. At many universities, the exams are held at the end of the term in a period of a few weeks after the end of the lectures. Again, there is also an optimization variant.

Both mentioned problems are NP(O)-complete and, hence, no polynomial time (exact) algorithms are known. As each university is faced with the ETTP and the CTTP several times per year, many different approaches are proposed in the literature to solve these problems automatically.

For the CTTP, there are two different variants presented in the literature. The first generates the timetable based on the information of the curricula and needs not to have students registered for courses, called the *Curriculum-based* course timetabling problem (CB-CTTP). The second variant needs to have the students registered for courses and can use this information in the timetabling algorithm. This is called the *Post Enrollment* course timetabling problem (PE-CTTP). Almost all literature defines the ETTP as a post enrollment problem and thus needs to have students registered for exams. With the ETTP being NP(O)-complete, the vast majority of algorithms developed for the examination timetabling problem are based on heuristics (e. g., see [8, 3,4]). A comprehensive extensive overview can be found in [10]. Other approaches are hyper heuristics, where, e. g., heuristics are chosen with the help of high level heuristics in order to explore the space of heuristic methods instead of the solution space. See [9] for an overview.

Caused by the "post enrollment" approach, when students register for an exam, they do not know at what date and time their exams will be held, which leads to animosity from the student body. To the best of our knowledge, almost all studies regarding the ETTP in the literature describe the problem as a post enrollment variant, with only a few exceptions ([1] and [2]).

In examination timetabling we deal with a large number of exams that need to be held in a relative short time period in only a small set of rooms. Different from course timetabling, in examination timetabling not every room of the institution can be used for exams, and the usable rooms have much less capacity as when used for a lecture due to an increase in required space between the students.

Furthermore compared to course timetabling, a conflict that could not be resolved between two exams is far more problematic for a student in reality, as for a course's

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lectures quite often students can use course material to not be actually present in the actual lecture. But for an exam it is physically impossible to take two exams at the same time.

In the literature most research is done on the post enrollment variant of examination timetabling. In this variant we have the exact information on how many students will participate in a given exam. We can also calculate what exams are actually in conflict simply on the fact that the same student takes both exams.

At the School of Engineering at the University of Erlangen-Nuremberg examination timetabling is done manually shortly before the end of the term when all students have registered for their exams. Presently, the examination timetabling is a two stages process. After all students have registered for their exams, a week and a day is assigned to each exam based on experience of the person responsible. Due to the rather small number of rooms available for all exams of the whole School of Engineering, the exact time and the rooms assigned to a particular exam can only be calculated and released a few days before the exam is held as the number of registered students drops again over time. This leads to animosity among the student body and the lecturers.

In this paper, we develop a curriculum-based variant of the examination timetabling problem (Sec. 2) and deliver a linear programming formulation (Sec. 3). The goal is to generate an examination timetable already at the beginning of the term without the need for human expert knowledge. We determine room assignments having sufficient capacity with probability as high as possible, based on data from previous terms and the number of enrolled students in the curricula. To improve the information provided by the curriculum, we use information obtained in previous terms and further integrate this knowledge into the model as a soft constraint. In our model an exam can use more than one room, further distinguishing our model from the common used problem instances provided in the ITC2007 exam timetabling track, which are widely used as benchmark functions in the literature. The results of an experimantal evaluation are presented in Sec. 4.

2 Probabilistic curriculum-based examination timetabling (PCB-ETTP)

Having an examination timetable at the beginning of the term at a university without the requirement for students to register for exams or even the lectures at the beginning is a strong argument for a variant of the examination timetabling that is based on curriculum information. The drawback is that when calculating the examination timetable, no exact number of students per exam is available and furthermore conflicts for exams can only be derived from the curriculum information, which may allow for many electives. The direct result is that how many students will take an exam needs to be estimated based on the curriculum information, and as such this number is in practice overestimated quite a bit. Estimating the number of students and the resulting conflicts for exams that are mandatory in a major is simple as every student in the corresponding grade must take all mandatory exams in the given grade. The problem comes from the elective lectures of a major available to students in a range of grades. Some lectures are more popular than others and as such the upper limit of students for this exam is the sum of all students in all grades the lecture can be chosen. As the curriculum information has no measure of popularity all elective exams need to be overestimated to a great extend making the resulting instance hard if not impossible to solve.

Each term such a derived instance needs to be solved. For the purpose of managing grades all students taking an exam have to be registered at some point and this information needs to be stored. The aim of our new model is to use this information and incorporate it to combine the good aspects of post enrollment examination timetabling and curriculum-based examination timetabling whilst negating the negative aspects of overestimation. In the design of our model for the PCB-ETTP we had two goals. The first goal is to stay as close to the existing models for the ETTP especially as described in [6] giving the possibility of using existing algorithms to solve the resulting problem instance. This enables the possibility of comparing our approach to existing results. The second goal was to integrate the available information to give estimations on the quality of the solution with respect to the uncertainties present in curriculum-based timetabling.

Now we describe the model of the PCB-ETTP.

An instance I of the Probabilistic Curriculum-based Examination Timetabling Problem (PCB-ETTP) is given as follows.

- A set $\mathscr{E}$ of exams
- A set $\mathscr{R}$ of rooms, together with a given capacity $\kappa : \mathscr{R} \to \mathbb{N}$
- Curricula C_s of the terms $s \in \{0, \ldots, S\} = \mathscr{S}$ with 0 being the term to be planned and the higher the number the older the term ($\mathscr{S}^+ = \{1, \ldots, S\}$)
  - A set $\mathscr{M}_{C_s}$ of major and grade combinations containing all available majors with all grades that have students associated
  - Number of students in major and grade $\nu_{C_s} : \mathscr{M}_{C_s} \to \mathbb{N}$
  - Associated exams for major and grade $\rho_{C_s} : \mathscr{M}_{C_s} \to \mathscr{P}(\mathscr{E})$
- A set of soft constraints with associated weights
- The number of students registered per exam in previous terms: $\nu_r : \mathscr{E} \times \mathscr{S}^+ \to \mathbb{N}$
- The number of students actually attending the exam in previous terms: $\nu_a : \mathscr{E} \times \mathscr{S}^+ \to \mathbb{N}$. Hence, $\nu_r(e,1) - \nu_a(e,1)$ is the number of students who withdrew from e in the previous term.
  Note that ν_r and ν_a are not defined for $s = 0$, i. e., the term to be planned.

To be feasible, a timetable must meet the hard constraints:

1. (H1) each exam is assigned to exactly one timeslot
2. (H2) each exam is assigned to one or more rooms
3. (H3) two exams associated to the same major and grade are not scheduled at the same time
4. (H4) no room is used at the same time by two exams
5. (H5) the sum of capacities of the assigned rooms is larger than the number of students taking the exam

Most of this model is similar to the model in [6]. However, the main contribution of the new model is that the number of students taking an exam in the current term is not yet specified in I. A simple approach to estimate the number of students taking an exam would be to just use the number of students taking the exam in the previous term or the arithmetic mean over the previous terms. This approach has one major drawback as it ignores changes in the curriculum itself. If the lecture can be chosen by another major starting this term or the number of students increases – as experienced in the popularity of computer science over the previous years – the number of students taking the exam can increase drastically. This change will be ignored in the mentioned approach and thus the assigned rooms might be too small for the exam. For the most part the popularity of a lecture does not change much over the course of a few terms. The same holds true for the number of students canceling the registration for an exam.

Definition 1 (Estimated number of students) *Let $e \in \mathscr{E}$ be an exam to be scheduled with* E[%register] *and* E[%withdraw] *the expected percentages of students registering for and withdrawing from exam e over the empirical distributions* %register *and* %withdraw *calculated from the previous S terms, and let σ be denoting the corresponding standard deviations. Furthermore let μ denote a safety factor to account for exams with a bigger fluctuation of students. $\psi(e,s) = \sum_{m:e\in\rho_{C_s}(m)} \nu_{C_s}(m)$ denotes the number of students in the associated majors of exam e.*

The estimated number of students taking exam e is

$$\mathrm{E}[\nu(e)] = \left\lceil \Big(\mathrm{E}[\%\text{register}] \cdot (1 - \mathrm{E}[\%\text{withdraw}]) + \mu \cdot \sigma[\%\text{register}]\Big) \cdot \psi(e,0) \right\rceil . \quad (1)$$

In the following, we treat $\mathrm{E}[\nu(e)]$ as an expected value. Instead of using the number of enrolled students directly, we calculate the factor of students registering in contrast to the maximum number of students possible and the factor of how many of these students withdraw from their registration. For each exam e we first calculate the empirical distribution values $\frac{\nu_r(e,s)}{\psi(e,s)}$ and $\frac{\nu_a(e,s)}{\nu_r(e,s)}$ over the terms $s \in \{1,\ldots,S\}$ and then take the average over these values to calculate the estimated number of students for this exam. By this calculation, a change in the curriculum is directly integrated into the timetabling process.

Given the uncertainty in the number of students taking an exam, the capacity constraint H5 is treated as a soft constraint with a high priority. In our model the number of students in an exam e is a random variable with the estimated (expected) number of students calculated as defined in Eq. 1. As more than one room can be assigned to an exam, we can formulate the hard constraint H2 such that an exam e needs to be assigned to a room pattern $p(e) \in \mathscr{P}(\mathscr{R})$ (here, $\mathscr{P}(\mathscr{R})$ denotes the powerset of $\mathscr{R}$). The total capacity of $p(e)$ is $\phi(p(e)) = \sum_{r\in p(e)} \kappa(r)$.

Fact 2 (Room pattern probability) *Let $e \in \mathscr{E}$ be an exam and let $p(e) \in \mathscr{P}(\mathscr{R})$ denote a room pattern with total capacity $\phi(p(e))$ With Markov's inequality we get:*

$$\Pr[\nu(e) > \phi(p(e))] \leq \frac{\mathrm{E}[\nu(e)]}{\phi(p(e))} \quad (2)$$

Note that the upper bounds as specified in Fact 2 use the *estimation* of the expected number of students attending the exam. As the exact distribution is not known and is only empirically calculated using the values of the previous years, the quality or even the validity of the upper bound cannot be specified. However, the better the estimation, the better the quality of the upper bounds. Furthermore, we only use this bound of $\frac{\mathrm{E}[\nu(e)]}{\phi(p(e))}$ as an estimation for the relative quality of room pattern p when assigned to exam e. The less this ratio, the better the pattern. Given the room pattern quality ratios, we can define an additional soft constraint for the PCB-ETTP.

Definition 3 $\max\left\{\frac{\mathrm{E}[\nu(e)]}{\phi(p(e))} \mid e \in \mathscr{E},\ p(e)\ \textit{is room pattern of e in a}\ \text{feasible}\ \textit{timetable}\right\}$ *should be minimal.*

Most soft constraints as defined in [6] directly influence the quality of the timetable with respect to the students and lecturers. However, this new soft constraint does not influence the perceived quality of the timetable directly, but the likelihood of a reschedule after students registered for the exams. By minimizing the quality ratio (probability) for an exam to be scheduled in rooms with insufficient total capacity, the actual quality of the upper bound does not influence the soft constraint as the error is applied to all possible room assignments for an exam.

3 Linear programs for the PCB-ETTP

For a more detailed description of this new model introduced in Sec. 2, we now provide a mathematical view in form of a linear program (LP). The formulation provided is intended to give a simple view of the new soft constraint and the overall model similar to [6].

3.1 An LP Model for the PCB-ETTP

In the description of the linear program used to implement our model, the timeslots over the course of the examination period are divided into weeks, days and periods with each week having a fixed number of days and a day is divided into a fixed number of periods, with each period long enough to hold any exam. The used sets in the description of the linear program are defined analogously to our model.

- $\mathscr{E}$: A set of exams to be planned
- $\mathscr{R}$: A set of rooms
- $W \subseteq \mathbb{N}$: A set of weeks
- $D \subseteq \mathbb{N}$: A set of days per week
- $P \subseteq \mathbb{N}$: A set of periods per day
  Hence, $(w,d,p) \in W \times D \times P$ is a *timeslot*.
- $\mathrm{E}[\nu(e)]$: The expected number of students taking exam $e \in \mathscr{E}$ as defined in Def. 1
- $\rho_{C_0} : \mathscr{M}_{C_0} \to \mathscr{P}(\mathscr{E})$: The function ρ_{C_0} associating exams to be planned with major and grade combinations in $\mathscr{M}_{C_0}$ contained in curriculum C_0
- $\mathscr{K} = \{(e_1,e_2) \in \mathscr{E} \times \mathscr{E} \mid e_1 \neq e_2, \exists m \in \mathscr{M}_{C_0} : (e_1,e_2) \in \rho_{C_0}(m) \times \rho_{C_0}(m)\}$: The set of conflicting exams in curriculum C_0

3.1.1 Variables

As the number of room pattern used in our model is exponential in the number of rooms we quantify over rooms instead of room pattern to reduce this exponential blow up.

$$X_{e,r}^{w,d,p} = 1 :\Longleftrightarrow \begin{array}{l}\text{Exam } e \in \mathscr{E} \text{ takes place in room } r \in \mathscr{R} \text{ and is scheduled} \\ \text{in timeslot } (w,d,p),\ w \in W,\ d \in D,\ p \in P\end{array}$$

For a more compact representation of the linear program, we introduce the secondary decision variables $h_{e,r}$ and $h_e^{w,d,p}$:

- $h_{e,r} = 1 :\Longleftrightarrow$ exam $e \in \mathscr{E}$ uses room $r \in \mathscr{R}$
- $h_e^{w,d,p} = 1 :\Longleftrightarrow$ exam $e \in \mathscr{E}$ is in timeslot (w,d,p)

To connect the secondary decision variables to the primary decision variables, we add the following constraints to the linear program.

- $h_e^{w,d,p} = 1 :\Longleftrightarrow$ Exam e is scheduled in timeslot (w,d,p).

  $$\forall e \in \mathscr{E}, w \in W, d \in D, p \in P:\ h_e^{w,d,p} \leq \sum_{r \in \mathscr{R}} X_{e,r}^{w,d,p} \leq |\mathscr{R}| \cdot h_e^{w,d,p} \tag{3}$$

  As more then one room can be assigned to exam e the second inequality forces $h_e^{w,d,p}$ to be 1 if exam e is in room r in timeslot (w,d,p), and the first inequality forces $h_e^{w,d,p}$ to be 0 otherwise.
- $h_{e,r} = 1 :\Longleftrightarrow$ Exam e uses room r in *some* timeslot (w,d,p)

  $$\forall e \in \mathscr{E}, r \in \mathscr{R}:\ h_{e,r} = \sum_{w \in W} \sum_{d \in D} \sum_{p \in P} X_{e,r}^{w,d,p} \tag{4}$$

3.1.2 Constraints

There are two types of constraints used in our model of the PCB-ETTP. The hard constraints that have to be satisfied in a feasible timetable and the soft constraints that establish the objective function describing the quality of a feasible timetable.

Hard constraints As defined in the PCB-ETTP model introduced in Sec. 2 for a timetable to be feasible each exam needs to be assigned to exactly one timeslot and one or more rooms.

- Exam e is assigned to exactly one timeslot. An exam will only be held once per term.

  $$\forall e \in \mathscr{E}:\ \sum_{w \in W} \sum_{d \in D} \sum_{p \in P} h_e^{w,d,p} = 1 \tag{5}$$

- Every exam needs to have enough rooms assigned to accommodate all students in the exam. S_e is set to the sum of the capacities of all rooms assigned to exam e and therefore needs to be greater than the estimated number of students taking exam e and $\kappa(r)$ is the size of room r.

  $$\forall e \in \mathscr{E}: S_e = \sum_{r \in \mathscr{R}} h_{e,r} \cdot \kappa(r) \tag{6}$$

  $$\forall e \in \mathscr{E}: \mathrm{E}[\nu(e)] \leq S_e \tag{7}$$

Equation 5 ensures that hard constraint H1 is satisfied. Furthermore exam conflicts and room restrictions have to be satisfied for a timetable to be feasible.

- Hard constraint H3 specifies that two conflicting exams must not be assigned to the same timeslot. Two exams are in conflict if they are associated to the same major and grade combination and therefore can be taken by the same student according to the curriculum information.

$$\forall m \in \mathscr{M}_{\mathscr{C}_0}, w \in W, d \in D, p \in P : \sum_{e \in \rho_{C_0}(m)} h_e^{w,d,p} \leq 1 \tag{8}$$

- No two exams can be assigned to the same room at the same time.

$$\forall r \in \mathscr{R}, w \in W, d \in D, p \in P : \sum_{e \in \mathscr{E}} X_{e,r}^{w,d,p} \leq 1 \tag{9}$$

Soft constraints In our linear program we use the soft constraints *two-in-a-row*, *two-in-a-day* and *period-spread* as defined in [6] modified to our curriculum model with the addition of the new soft constraint as defined in Def. 3. As we describe our linear program as a minimization problem the minimization of the variables $V^{2\mathrm{R}}$, $V^{2\mathrm{D}}$ and V^{PS} will try to set the variables $V^{2\mathrm{R}}_{(e_1,e_2)}$, $V^{2\mathrm{D}}_{(e_1,e_2)}$ and $V^{\mathrm{PS}}_{(e_1,e_2)}$ of a conflict (e_1, e_2) to 0 and the variables will only be forced to 1 if there is a violation of the corresponding soft constraint for the conflict. If there is a violation of the soft constraints *two-in-a-row*, *two-in-a-day* and *period-spread* the minimum of the students attending both exams will be added to the objective as a penalty. For the new soft constraint instead of quantifying over room patterns for the new soft constraint, we quantify over all possible room pattern sizes as the ordered set of increasing sizes $\mathscr{G}$.

- If two exams are conflicting according to the curriculum C_0 they should not be assigned to two adjacent timeslots on the same day.

$$\begin{aligned}\forall (e_1, e_2) \in \mathscr{K}, w \in W, d \in D, \forall p, q \in P, |p-q| = 1 :& \\ h_{e_1}^{w,d,p} + h_{e_2}^{w,d,q} \leq 1 + V^{2\mathrm{R}}_{(e_1,e_2)}&\end{aligned} \tag{10}$$

$$V^{2\mathrm{R}} = \sum_{(e_1,e_2) \in \mathscr{K}} \min\{\mathrm{E}[\nu(e_1)], \mathrm{E}[\nu(e_2)]\} \cdot V^{2\mathrm{R}}_{(e_1,e_2)} \tag{11}$$

- If two exams are conflicting according to the curriculum C_0 they should not be assigned to two timeslots on the same day.

$$\begin{aligned}\forall (e_1, e_2) \in \mathscr{K}, w \in W, d \in D, \forall p, q \in P, |p-q| \geq 2 :& \\ h_{e_1}^{w,d,p} + h_{e_2}^{w,d,q} \leq 1 + V^{2\mathrm{D}}_{(e_1,e_2)}&\end{aligned} \tag{12}$$

$$V^{2\mathrm{D}} = \sum_{(e_1,e_2) \in \mathscr{K}} \min\{\mathrm{E}[\nu(e_1)], \mathrm{E}[\nu(e_2)]\} \cdot V^{2\mathrm{D}}_{(e_1,e_2)} \tag{13}$$

– If two exams are conflicting according to the curriculum C_0 they should not be assigned to two timeslots in a range of λ.

$$\forall (e_1,e_2) \in \mathscr{K}, w \in W, \forall d_1,d_2 \in D, \forall p,q \in P,$$
$$1 \leq |(d_1 \cdot |P| + p) - (d_2 \cdot |P| + q)| \leq \lambda :$$
$$h_{e_1}^{w,d,p} + h_{e_2}^{w,d,q} \leq 1 + V_{(e_1,e_2)}^{\mathrm{PS}} \tag{14}$$

$$V^{\mathrm{PS}} = \sum_{(e_1,e_2) \in \mathscr{K}} \min\{\mathrm{E}[\nu(e_1)], \mathrm{E}[\nu(e_2)]\} \cdot V_{(e_1,e_2)}^{\mathrm{PS}} \tag{15}$$

– We define the indicator variables $P_{e,g}$ with $P_{e,g} = 1 :\Longleftrightarrow$ exam e is assigned to room pattern of size g. Therefore only one $P_{e,g}$ can be 1 and the variable corresponding to the rooms assigned to exam e is set to 1. Note that two room patterns with the same size have the same associated probability for the same exam as defined in Fact 2.

$$\forall e \in \mathscr{E} : \sum_{g \in \mathscr{G}} P_{e,g} = 1 \tag{16}$$

$$\forall e \in \mathscr{E} : S_e - \sum_{g \in \mathscr{G}} P_{e,g} \cdot g = 0 \tag{17}$$

– With the variables $P_{e,g}$ we can set P_e to the upper bound of the probability for exam e to have a room pattern, with not enough capacity as defined in Def. 2 and $P^{\max}$ as the maximum over all P_e.

$$\forall e \in \mathscr{E} : P_e = \sum_{g \in \mathscr{G}} P_{e,g} \cdot \frac{\mathrm{E}[\nu(e)]}{g} \tag{18}$$

$$\forall e \in \mathscr{E} : P_e - P^{\max} \leq 0 \tag{19}$$

Note that the soft constraint 18, 19 are not integer constraints. The size of the linear program is in $O(|\mathscr{M}_{C_0}| \cdot |W| \cdot |D| \cdot |P| + |\mathscr{E}| \cdot |\mathscr{G}|)$ and in practice smaller than a linear program using room patterns directly.

3.1.3 Objective

The objective of the linear program is simply to minimize the weighted sum of the soft constraints penalties.

$$\text{minimize } \alpha \cdot V^{\mathrm{2R}} + \beta \cdot V^{\mathrm{2D}} + \gamma \cdot V^{\mathrm{PS}} + \delta \cdot P^{\max} \tag{20}$$

Table 1: Data sets provided by the School of Engineering at the University of Erlangen-Nuremberg

	Winter term	Summer term
Exams	168	157
Rooms	13	13
Conflicts	4966	3780
Curricula and Grades	908	819
Students	7984	7019
Timeslots	5 weeks, 5 days, 3 periods	

4 Experiments

At the School of Engineering at the University of Erlangen-Nuremberg exams are held at the end of a term, while all lectures are suspended. Almost all lectures are held in either the winter or the summer term, with the main exam at the end of the corresponding term and an exam for students that failed or canceled the main exam at the end of the next term. Therefore we created two distinct exam data sets one for the summer and one for the winter term, for a detailed description of the data sets see Table 1. We generated two PCB-ETTP instances, one for the summer term with training data taken from the years 2014–2016 and curriculum data from the summer term 2017 and one for the winter term with training data taken from the years 2014–2015 and curriculum data from the winter term 2016. We then compared the two PCB-ETTP instances against the corresponding registration data.

4.1 Estimation of the expected students number

To generate the PCB-ETTP instance we first have to calculate the empirical enrollment and cancel factor distributions. For each exam and term we calculate these factors with data taken from the curriculum information and the actual registration data. If we compare the estimated number of students calculated for the summer term 2017 and winter term 2016 with the number of students actually taking the exam in the corresponding terms we get the absolute errors for each exam. Tables 2 and 3 show the median positive and negative errors for each term for different values of the safety factor μ used in the estimation of the number of students taking an exam in Def. 1.

As shown in the Tables 2 and 3, the median error in the estimation of the number of students taking an exam is low with a somewhat higher relative error. The higher relative error comes from the elective exams as they often have only a few students attending. Therefore a slight absolute error in the estimation can lead to a big increase in the relative error. This difference however has no significant impact on the resulting timetable as theses exams mostly need only one room and the rooms assigned have enough slack to compensate the error in the estimation. We can observe that with increasing safety factor μ the number of exams that have an positive error, i.e., are overestimated, increases but the overall error itself increases significantly while the negative errors only decrease slightly.

Table 2: Median positive and negative absolute errors for the summer and winter term with the corresponding number of exams having a positive or negative error. μ denotes the safety factor.

	Summer term		Winter term	
	error / values		error / values	
μ	positive	negative	positive	negative
0	13 / 105	6 / 52	11 / 101	12 / 67
0.1	15 / 111	6 / 46	12 / 106	12 / 62
0.3	17 / 121	6 / 36	15 / 117	10 / 51
0.5	19 / 129	9 / 28	16 / 127	10 / 41
0.7	21 / 135	8 / 22	18 / 131	11 / 37
1	25 / 141	5 / 16	23 / 136	10 / 32

Table 3: Median positive and negative relative errors for the summer and winter term with the corresponding number of exams having a positive or negative error. μ denotes the safety factor.

	Summer term		Winter term	
μ	positive error	negative error	positive error	negative error
0	0.382	0.209	0.273	0.194
0.1	0.413	0.186	0.300	0.172
0.3	0.471	0.200	0.341	0.169
0.5	0.529	0.185	0.383	0.156
0.7	0.588	0.182	0.450	0.155
1	0.686	0.184	0.533	0.148

4.2 Impact of the estimation errors

Given the rather small errors between the estimated and the actual number of students taking an exam the new soft constraint should be sufficient to compensate and help influence the calculation of the timetable to produce a result where every exam can accommodate all students taking the exam. To test this hypothesis the PCB-ETTP instance with $\mu = 0.1$ for the summer term 2017 was used to generate a linear program as specified in 3. This linear program was solved by CPLEX [5] and a non optimal solution was returned due to an upper bound on the iterations. In our test the resulting timetable could accommodate all students and no reschedule would have been necessary.

4.3 Influence of the proposed soft criterion

To test the influence of the new soft criterion on the overall quality of the resulting timetable we reduced our instance to 25 exams as the CPLEX solver was able to solve the generated instances in a time frame of 2 hours returning the optimal solution. For this test we generated 20 such instances for each we sampled u.a.r. 25 exams from the summer term 2017 instance and further reduced the number of rooms to 7 and the number of days to 3, periods to 3 and weeks to 1. In the tests we used the soft constraint weights 20 for C^{2R}, 5 for C^{2D} and 1 for C^{PS} with $\lambda = 4$. For

Table 4: Average absolute error of C^{2R}, C^{2D}, C^{PS} when compared with the respective reference solution and the average values of P^{max} for 20 different runs

	reference solution	$500 \cdot P^{max}$	$1000 \cdot P^{max}$	$1500 \cdot P^{max}$
C^{2R}	41.7	0.05	-0.6	-0.2
C^{2D}	194.1	-0.05	3.15	2.75
C^{PS}	897.6	12.25	35.1	36.5
P^{max}	0.98478353	0.6431269	0.6050174	0.59879196

each generated instance we performed 3 optimization runs with the weights 500, 1000 and 1500 for P^{max} respectively and as a reference solution we performed one optimization without the proposed soft criterion. Table 4 summarizes the results of the different optimization runs. Using the new soft criterion the quality of the returned solution decreases only slightly in comparison to the reference solution as shown in Table 4. For the chance of a reschedule after registration however we get a drastic improvement in respect to the reference solution. One can argue that the solution when using the proposed soft criterion uses more rooms than necessary for exams. In practice however not every room associated to the exam needs to be used and can be adjusted by the lecturer after the registration or prior in a cleanup phase.

5 Conclusion

In this work we introduced a new model for the examination timetabling problem. Instead of calculating the timetable based on enrollment data, as is done in the majority of the literature, we generate the data from curriculum information and use data from previous terms to improve the estimation on how many students take an exam. Through this technique we could show that the estimations deviate only by a small amount from actual registration data as shown for the last two terms at the School of Engineering at the University of Erlangen-Nuremberg. Furthermore we introduced a new soft constraint to minimize the impact of the remaining deviation of the student data when calculating the timetable for the exams. Through this method we introduced a quality measure for the result of the calculation enabling users to balance between minimizing the risk of having to change times of the exams and the other quality measures regarding acceptance from students. In our tests we could show that the quality of the solution decreases only by a small amount when using the proposed soft criterium. The model was kept close to existing models of the examination timetabling problem enabling existing algorithms to also use this model with only small changes.

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