Flexible shift scheduling of healthcare workers using branch and price

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Abstract. We consider a strategic shift scheduling problem, consisting of assigning health care workers in a hospital to shifts for handling a given number of tasks spread out over a fixed time horizon. We formulate and present a branch-and-price algorithm with a novel network flow formulation for the subproblem.

Keywords: Branch-and-price, shift scheduling, shortest path

1 Branch-and-price

Most shift scheduling and rostering problems in literature are solved using decomposition algorithms, Mixed Integer Programs (MIPs), e.g. Brunner et. al. [3] or heuristics, e.g Van Huele and Vanhoucke [1], as presented by Van den Bergh et. al. [2]. We will look into an extension of decomposition of a MIP model.

We will implement a branch-and-price algorithm with the master problem assigning schedules to workers and the subproblem generating columns (shift schedules) to add to the master problem. The master problem will minimize the total cost of the chosen shift schedules and assign outside workers to handle understaffing. The novelty in our approach lies in the specific formulation of the network flow model for schedule generation. This is fairly well-known in literature, e.g. Akbarzadeh and Maenhout (2021) [5] use such an approach for scheduling medical students and . Instead of formulating the subproblem as a MIP, we will make use of a novel network formulation. We define a network first introduced [4] By traversing the network from the starting period to the end, a path, corresponding to a valid shift schedule, will be found. As such, the subproblem can be solved, and a maximum reduced cost schedule can be found by solving the shortest path problem for the graph. We will implement Dijkstra's algorithm with a labelling and readout step to find the shortest path through the network.

As branching strategies we wish to test two separate approaches. The first is approach is to branch on the most fractional masterproblem variable $\lambda_j = \lambda_j^*$, by setting upper and lower bounds in the left and right branch respectively. This is easily included by fixing the value in each iteration of the master problem. However the in the left branch to ensure that the schedule is not regenerated, we solve a k-shortest path problem and compare the generated schedules with all schedules in the current masterproblem. This is equivalent to introducing schedule elimination constraints in an IP version of the subproblem [6].

The second strategy is to branch on the most fractional set of working periods, by restricting the usage of all schedules using these working periods, a strategy used in

[4]. This can be implemented by a restriction in the masterproblem and by restricting this combination of working periods in the network, e.g. via removing weights and generating a set of shortest paths. This way all the generated schedules will adhere to the imposed restrictions. At each node of the branching tree, the lower bound is found via solving the LP relaxation of the MP to optimality. The upper bound is found via a simple rounding heuristic that fixes a single employee to a specific schedule and covers the remaining demand via external staff. This is used as the computational cost is low. A more precise upper bound is found by solving the MP as an IP problem, but due to the higher computational cost, this is only done at certain intervals.

Any under coverage resulting from these schedules will be handled by adding external staff in the MP.

We will look into the possibility of scheduling flexible and in-flexible workers to achieve a greater flexibility in the generated schedules. The fraction of flexible workers will be evaluated with varying proportions to ensure a feasible application of the method. The extended branch-and-price method will be applied and tested on a number of test instances generated from real-world data.

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