# The University Examination Timetabling Problem with Uncertain Timeslot Capacity: A Two-stage Stochastic Programming Approach

Sara Ceschia[1] [0000−0003−1191−1929], Daniele Manerba[2] [0000−0002−3502−5289], Andrea Schaerf[1] [0000−0001−6965−0536], Eugenia Zanazzo[1], and Roberto Zanotti[3] [0000−0002−3073−4895]

[1] Polytechnic Dept. of Engineering and Architecture, Università di Udine, Italy
{sara.ceschia,andrea.schaerf,eugenia.zanazzo}@uniud.it
[2] Dept. of Information Engineering, Università degli Studi di Brescia, Italy
daniele.manerba@unibs.it
[3] Dept. of Clinical and Experimental Sciences, Università degli Studi di Brescia, Italy
roberto.zanotti@unibs.it

**Abstract.** In this work, we extend the original Uncapacitated Examination Timetabling problem by introducing capacity constraints that limit the number of exams schedulable per timeslot and, to take into account possible unexpected disruptive events, by considering such a capacity as a random variable. We propose a two-stage Stochastic Programming approach for this stochastic variant in which recourse actions allow rescheduling exams in successive timeslots or moving students to *spot-market* rooms. Then, we conduct an in-depth analysis of the impact of uncertainty on solutions using a deterministic equivalent Mixed-Integer Linear Programming formulation. Additionally, we plan to develop a Progressive Hedging algorithm, leveraging the efficiency of a specialized optimizer [4], to address the computational challenges posed by the stochastic nature of the problem even for small-medium size instances. Preliminary results are promising, underscoring the significance of accounting for stochasticity in the problem formulation.

**Keywords:** Examination timetabling, Uncertain timeslot capacity, Two-stage Stochastic Programming with recourse

## 1 Introduction

In the context of university organization, the Examination Timetabling problem (ETT) aims at assigning exams to timeslots ensuring that *i*) each exam is scheduled exactly once during the examination period, *ii*) two conflicting exams are not scheduled in the same timeslot, and *iii*) the total penalty associated with the created timetable is minimized [6]. Among many existing formulations, the most classic, known as Uncapacitated ETT (UETT), was introduced in [5] and specifically penalizes exams with students in common scheduled within a distance less than or equal to 5 timeslots.

In the UETT, it is assumed that the number of exams scheduled in each timeslot is unbounded. However, in practical applications, physical constraints (number of available rooms and their capacity) must be taken into consideration. In addition, after the exam calendar is released, uncertain events may occur and reduce the day-by-day availability of resources (rooms, teachers, timeslots), thus making the original schedule infeasible. To address this issue, we investigate a Stochastic Capacitated ETT under uncertain timeslot capacity (S-CETT), in which the number of exams schedulable in each timeslot is modeled using a random variable. In particular, we formulate the S-CETT as a two-stage Stochastic Programming (SP) problem and study the impact of uncertainty on timetables given the implementation of reasonable recourse actions. Finally, we plan to implement an efficient algorithm approach hybridizing an SP decomposition-based matheuristic and a tailored state-of-the-art heuristic method.

To our knowledge, little attention has been paid to uncertainty in the context of ETT problems (see [2,3]). Instead, the problem of finding a robust timetable for the Curriculum-Based University Course Timetabling problem subject to different types of disruptions has been addressed in [1,7,10]. Such a problem is usually modeled as a minimum perturbation problem with a bi-criteria objective function, where the first objective is related to the quality of the solution and the other is about the robustness of the timetable.

## 2 ILP formulation for the S-CETT

Let us consider a set $E$ of exams, to be scheduled during an examination period at the end of the semester, and a set $S$ of students. Each student is enrolled in a non-empty subset of exams. The examination period is divided into $T$ ordered timeslots, each having a scheduling capacity of $B$ exams. Let $n_e$ be the number of students enrolled in exam $e \in E$. Given two exams $e, e' \in E$, let $n_{e,e'}$ be the number of students enrolled in both. Two exams $e, e' \in E$ are called *conflicting* if they have at least one student enrolled in both, i.e., if $n_{e,e'} > 0$. Let us define the set $C$ of conflicts, including all the exam pairs $[e, e']$ with $e, e' \in E$ for which $n_{e,e'} > 0$. *Conflicting* exams cannot take place during the same timeslot. Moreover, to foster the creation of timetables that are more sustainable for the students, a penalty is assigned for each couple of conflicting exams scheduled up to a *distance* of 5 timeslots. More precisely, given two exams $e, e' \in E$ scheduled at distance $i$ of time-slots, with $1 \leq i \leq 5$, the relative penalty is $2^{(5-i)} n_{e,e'}$. Finally, let $\tilde{B}_t$ be a stochastic variable representing the loss of capacity of scheduled exams in timeslot $t = 1, \ldots, T$, with $0 \leq \tilde{B}_t \leq B$.

Let us define a binary variable $y_{e,t}$ determining the assignment of exam $e \in E$ to time-slot $t = 1, \ldots, T$, and binary variable $u^i_{e,e'}$ which takes value 1 if the conflicting exams pair $[e, e'] \in C$ is scheduled $i = 1, \ldots, 5$ time-slots apart, and 0 otherwise. Then, our S-CETT can be formulated as follows:

$$\text{(S-CETT)} \quad \min \quad \frac{1}{|S|} \sum_{i=1}^{5} \sum_{[e,e'] \in C} 2^{(5-i)} n_{e,e'} u^i_{e,e'} \tag{1}$$

subject to

$$\sum_{t=1}^{T} y_{e,t} = 1 \qquad e \in E \tag{2}$$

$$y_{e,t} + y_{e',t} \leq 1 \qquad [e, e'] \in C, t = 1, \ldots, T \tag{3}$$

$$y_{e,t} + y_{e',t+i} \leq 1 + u^{i}_{e,e'} \qquad [e,e'] \in C, i = 1,\ldots,5, t = 1,\ldots,T-i \tag{4}$$

$$\sum_{e \in E} y_{e,t} \leq B - \tilde{B}_t, \qquad t = 1,\ldots,T. \tag{5}$$

The objective function (1) minimizes the overall penalty by summing up individual penalties for each couple of conflicting exams. Constraints (2) ensure that each exam is scheduled exactly once. Constraints (3) ensure that two conflicting exams can not be scheduled in the same timeslot. Constraints (4) ensure that if two conflicting exams are scheduled $i$ timeslots apart (i.e., both the $y$ variables in the inequality take value 1), then the relative $u$ variable must take value 1 as well. Finally, constraints (5) ensure that the number of exams scheduled in a timeslot is limited by the stochastic capacity $B - \tilde{B}_t$.

## 3 SP framework and solution approach

We tackle the problem using a two-stage SP paradigm, in which first-stage variables concern the pre-scheduling of exams to timeslots. In contrast, the second-stage recourse actions include the possibility of *i*) rescheduling the exams in a different timeslot after the pre-scheduled one and *ii*) moving an exam to a *spot-market* room in the same timeslot. Note that it is possible to relocate any number of exams to the *spot-market* room. This way the model always guarantees a feasible solution. However, in addition to the basic penalties due to exam incompatibilities, extra penalties, proportional to the number of students affected by rescheduled exams or moved to the spot-market room, must be considered in the expected value.

To practically address the problem via state-of-the-art MIP solvers, we create a deterministic equivalent formulation by approximating the behavior of the random variables involved through a finite (but sufficiently large) number of future scenarios, each occurring with a given probability. This allows us to validate our model by assessing standard SP indicators, such as the *Value of the Stochastic Solution* (VSS) and the *Expected Value of the Perfect Information* (EVPI). VSS represents the penalty saving given by using our SP approach instead of a deterministic model, while EVPI represents how much we would be willing to pay for not having uncertain data. Figure 1 presents boxplots on the percentage values of these two indicators obtained on a set of 20 small instances, each with $|E| = 10$, $T = 7$, $B = 2$, and 20 scenarios. VSS and EVPI both have an average value of around 30%, indicating that there is a significant gain in accounting for stochasticity but also a notable gap from the case in which the values of all random variables are known beforehand. The VSS, ranging from 10 to 50%, particularly proves the importance of a more robust provisional schedule and more flexible recourse decisions.

In the same figure, we also show an additional indicator, named *Stochastic Loss* (SL), that measures the percentage difference between the first-stage penalty in our stochastic variant and the objective function value of the deterministic problem. This indicator shows that, on average, the solutions of our variant include a 65% higher penalty when it comes to the provisional schedule of the exams to be more conservative and handle the uncertainty of future events more effectively.

Apart from the above validation, the use of an exact technique can be computationally too expensive against real-size instances with a representative number of scenarios.

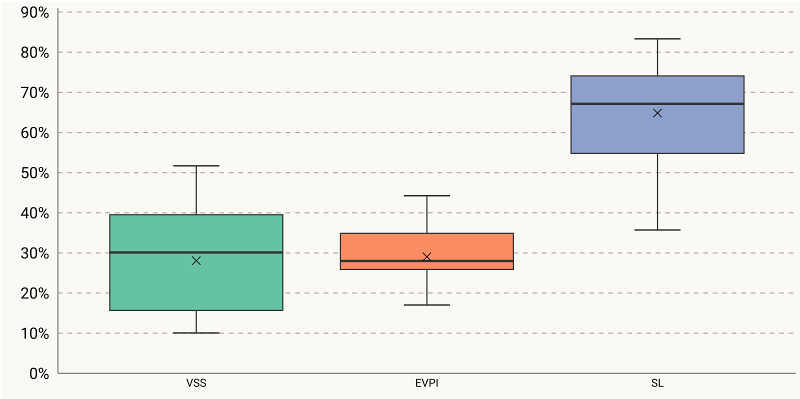


Fig. 1: Percentage values of VSS, EVPI, and SL over all the benchmark instances.

Hence, we plan to develop a heuristic convergence framework based on a Progressive Hedging (PH) algorithm ([8],[9]), which decomposes the problem per scenario and forces a *consensus* solution among the scenarios via an Augmented Lagrangian Relaxation approach. To solve the deterministic mono-scenario subproblems, iteratively created during the PH, we will use the Simulated Annealing-based algorithm developed in [4] for the UETT conveniently adapted to the capacitated version. Extensive computational results of the PH framework will be presented at the conference.

## References

1. Akkan, C., Gülcü, A., Kuş, Z.: Minimum penalty perturbation heuristics for curriculum-based timetables subject to multiple disruptions. Computers & Operations Research **132**, 105306 (2021)
2. Bassimir, B., Wanka, R.: Probabilistic curriculum-based examination timetabling. In: Proc 12th International Conference on the Practice and Theory of Automated Timetabling (PATAT). pp. 273–285 (2018)
3. Bassimir, B., Wanka, R.: Robustness approaches for the examination timetabling problem under data uncertainty. In: Proc. 9th Multidisciplinary International Scheduling Conference: Theory & Applications (MISTA) (2019)
4. Bellio, R., Ceschia, S., Di Gaspero, L., Schaerf, A.: Two-stage multi-neighborhood simulated annealing for uncapacitated examination timetabling. Computers & Operations Research **132**, 105300 (2021)
5. Carter, M.W., Laporte, G., Lee, S.Y.: Examination timetabling: Algorithmic strategies and applications. Journal of the Operational Research Society **47**, 373–383 (1996)
6. Ceschia, S., Di Gaspero, L., Schaerf, A.: Educational timetabling: Problems, benchmarks, and state-of-the-art results. European Journal of Operational Research **308**(1), 1–18 (2023)
7. Lindahl, M., Stidsen, T., Sørensen, M.: Quality recovering of university timetables. European Journal of Operational Research **276**(2), 422–435 (2019)
8. Løkketangen, A., Woodruff, D.L.: Progressive hedging and tabu search applied to mixed integer (0,1) multistage stochastic programming. Journal of Heuristics **2**(2), 111–128 (1996)

9. Manerba, D., Perboli, G.: New solution approaches for the capacitated supplier selection problem with total quantity discount and activation costs under demand uncertainty. Computers & Operations Research **101**, 29–42 (2019)
10. Phillips, A.E., Walker, C.G., Ehrgott, M., Ryan, D.M.: Integer programming for minimal perturbation problems in university course timetabling. Annals of Operations Research **252**, 283–304 (2017)