Performance impact of constraint variants on a MILP formulated hearing scheduling problem

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1 Introduction

The courts within the judicial system involve a labour-intensive service supply chain, where planners are faced with the challenge of maintaining accessibility for litigants while balancing the workload of court staff (clerks and judges).

Over the past 50 years, to the best of our knowledge, only 42 articles have been published that adopt operation research and management to court operations. One of the earlier papers,[3], researched the impact of organisational changes on delay for felony defendants through simulation. [2] proposed a two-step ILP model that calculates the optimal number of judges per district and distributes judges over districts by maximising the disposal of cases. [4] developed a strategic model to define court districts and the locations of courts within those districts. [1] proposed a model to schedule sessions and allocate judges to them by minimising the number of violations per judge. This paper is one of the few that shows similarities with our problem.

The central problem of this research is the combined decision of scheduling hearing blocks over time and the allocation of resources to these hearing blocks. When made correctly, the service and case mix the court aims to cover are met, aligning demand and supply. Currently, the court of law experiences difficulties in reaching the agreed case mix. Solving the central problem results in a hearing block schedule, which reserves sufficient capacity for court case groups. Given such a block schedule, cases can be assigned to these blocks in the subsequent operational Case Booking Problem (CBP). In this research, we only focus on the Hearing Scheduling Problem (HSP), part of the tactical level of planning problems in this service supply chain.

Figure 1 shows an example of a non-cyclic hearing block schedule. As can be found in the example, hearing blocks for specific case groups are scheduled on day-parts. These blocks require particular resources in terms of courtrooms and skilled staff members. Some blocks need three judges and one clerk; others need only one judge and one clerk, later referred to as multi-judge or single-judge blocks. Moreover, staff members are allocated to on-call duties required for urgent cases entering the court system.

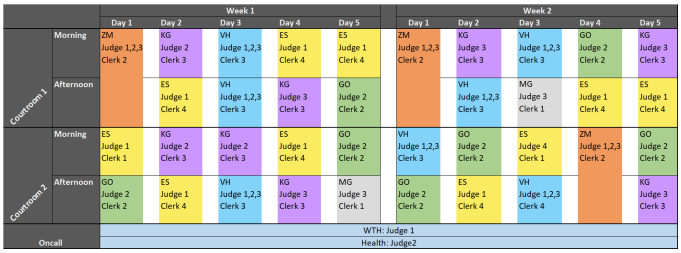

Fig. 1: Example of a hearing block schedule as used in the Court of Law

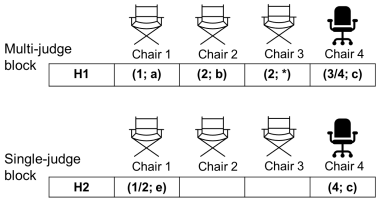

Fig. 2: Skill set (experience level; expertise area) required for chairs during a multi-judge or single-judge blocks.

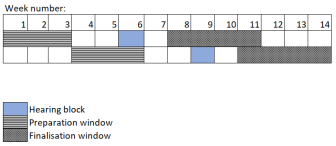

Fig. 3: Execution of desk time activities in time windows.

The capacity and skill set of the staff members determines the maximum number of scheduled hearing blocks. So, when generating the schedule, the assigned staff member must be available and have a suitable skill set. Each staff member has one experience level and could have multiple expertise areas. Figure 2 visualises that each chair part of a hearing block requires a specific combination of experience level and expertise area.

Highly interesting to this problem is the consumption of capacity by desk activities that prepare and finalise cases handled on the hearing blocks. When scheduling a hearing block, the chosen staff members must have sufficient capacity for preparation and finalisation in a specific time window. Figure 3 shows that a block scheduled in week 6 must be prepared in weeks 1 to 3 and finished in weeks 8 to 10. Depending on the case group, preparation and finalisation take between 4 and 10 hours. Desk time activities for different blocks can be parallel executed and staff members decide when this is done during the allowed window.

In the remainder of this extended abstract, we introduce two MILP variants for the desk time activities and we explain our experimental design, which analyses the computational performance of different formulations for the desk time assignment. We conclude with the variant that outperforms the other and further research steps.

2 Model formulation

In this section, we provide important parts of our MILP formulation for the HSP. We have a set of hearing block types $\mathcal{H}$, a set of chairs per type h $\mathcal{I}_h$, a set of courtrooms $\mathcal{R}$,

a set of day parts $\mathcal{D}$, and a set of staff members $\mathcal{S}$. In line with the proposed definition for the HSP, the following decisions are made:

$$X_{h,r,d} = \begin{cases} 1, & \text{if hearing block of type } h \text{ is assigned to courtroom } r \text{ on day part } d. \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_{s,h,i,r,d} = \begin{cases} 1, & \text{if staff member } s \text{ is allocated to slot } i \text{ in hearing block of type } h \\ & \text{scheduled in room } r \text{ on day part } d. \\ 0, & \text{otherwise.} \end{cases}$$

The primary objective of the HSP is to maximise the number of hearing blocks over the schedule horizon:

$$\max \sum_{h \in \mathcal{H}} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} X_{h,r,d}$$

The decisions are made under the condition that each staff member is assigned at most once per hearing block, courtroom, and day part combinations. Moreover, a hearing block can be allocated once per combination of a day part and courtroom, and only if courtroom r is available on day part d. As Section 1 explains, assigning a staff member with the appropriate skill set for a slot part of the hearing block is key. It is also important that, at most, one eligible staff member is assigned to a chair. At last, when the multi-judge block is required, all slots must be filled with the required staff members. We elaborate on the formulations of these restrictions in the full paper.

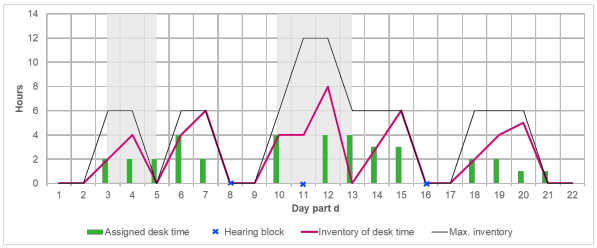

Fig. 4: Concept behind variant B for desk time activities

An interesting condition is the capacity consumption by desk activities associated with an assigned hearing block. We decide to formulate this in two different ways. Variant A assigns desk time in hours to a day part d for each scheduled hearing block without exceeding a staff member's capacity. Variant B is inspired by inventory balance equations and aggregates desk time into an inventory that is consumed when a desk activity needs to be finished. Figure 4 illustrates this concept. For a block scheduled on day part 8, inventory for preparation is increased within the allowed window from day part 3 till 5 and thereafter consumed on day part 5.

3 Solution method and experimental design

Our experiments focus on the computational performance of different formulations for desk time assignments, using generated instances based on real-life data. We generated 10 instances per instance class. Each class has a different schedule period, which increases from 3 to 15 weeks. A maximum of 15 weeks is chosen, because the court of law uses quarterly schedules. Between individual instances within classes, the staff member availability per day part is uniformly distributed between 2 and 4 hours. All the other parameters remain similar between individual instances and instance classes.

The MILP is implemented in AIMMS and solved by GUROBI 11.0.1 with a Lenovo Thinkpad with an Intel(R) Core(TM) i7-6700HQ CPU @ 2.600GHz 2.6 GHz and 16GB RAM CORE i7.

Weeks:	3	4	5	6	7	8	9	10	11	12	13
Variant A	0,28	6,68	2646,36	4740,81	7277,15	inf.	inf.	inf.	inf.	inf.	inf.
Variant B	0,00	4,17	4,37	4,14	3,30	2,60	2,11	6,69	1,56	1,31	1,35

Fig. 5: Integrality GAP after 300 sec.

Figure 5 compares the average integrality GAP after solving for 300 seconds between variant A and B. GUROBI cannot find a solution for variant A for instances larger than 7 weeks. Variant B outperforms variant A, since GUROBI finds a solution with a smaller integrality GAP for all instance classes. When comparing the variants, the difference in integrality gap can be declared by the strong increase in number of constraints and variables as shown in Figure 6.

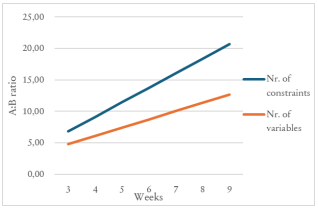

Fig. 6: Increase of ratio A:B

4 Conclusion and Further steps

Our contribution is two-fold: first, we introduced a MILP formulation of the underexposed HSP, and second, we analysed its computational performance under various constraint formulations. Our results show an improved computational performance when modelling desk time assignment as an inventory balance equation.

However, improvement of the modelling approach is still possible since the generated schedule shows an imbalanced spread over time in the hearing blocks scheduled per type. Therefore, our current steps focus on generating a schedule in which blocks are spread over the horizon. This is done by extending the objective with a secondary objective function incorporating a spread measurement. In our presentation, we will provide the MILP formulation more extensively and discuss the preliminary results of our next steps.

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