Novel neighborhood structures for incomplete round robin tournaments

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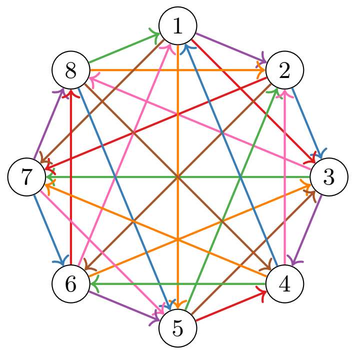
1 Sports timetabling as a graph problem

In this work, we study novel neighborhood structures for incomplete round robin (iRR) tournaments. In this tournament format, we are given a set of n teams and a set of r rounds, such that $r < n - 1$, i.e. not every team can face every other team. Hence, in contrast to round robin tournaments (RR), we have to determine the subset of games that is played. In graph theory terms, if we let teams correspond to vertices and matches correspond to edges, a conventional round robin tournament can be seen as a proper edge coloring of the complete graph K_n, where the color denotes the round in which a match is played. In contrast, in an incomplete round robin tournament, some of the edges are uncolored (meaning that these matches are not scheduled), and we have to find an r-regular subgraph of K_n that is properly edge colored with r colors, while all other edges in K_n remain uncolored. In this case, we refer to the coloring as a *partial proper edge coloring*. This difference is illustrated in Figures 1 and 2 for a RR and iRR tournament with 8 teams, respectively, where only 5 rounds are played in the iRR tournament.

2 Related literature

The iRR format has gained significantly in popularity since its introduction in the three major European football competitions, i.e. the UEFA Champions, Europa and Conference league, respectively [1, 3, 4, 8]. Moreover, it has also seen its introduction in youth sports competitions. In particular, it has been shown that the iRR format significantly reduces travel time compared to the format in which teams are partitioned into leagues and each league is organized as a double round robin tournament [2, 7]. Despite this gain in popularity, literature about neighborhood structures for iRR tournaments is sparse. Indeed, existing neighborhood structures for sports scheduling have so far been developed almost solely for round robin tournaments. Although their nomenclature differs throughout the literature, they are commonly known as Team Swap, Partial Team Swap, Round Swap, Partial Round Swap, and Cycle-and Path reversal [5, 6]. As a first contribution, we show that these neighborhood structures remain applicable for iRR tournaments.

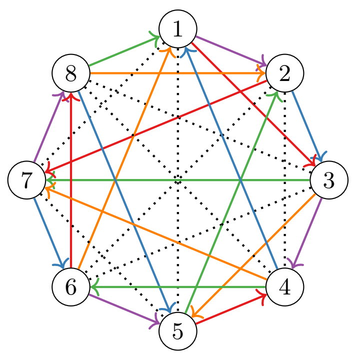


(a) Proper edge coloring of K_8

R1	R2	R3	R4	R5	R6	R7
3-1	3-2	7-3	4-3	3-6	8-3	3-5
7-2	1-4	2-5	2-1	2-8	2-4	6-2
8-6	5-8	1-8	8-7	5-1	1-6	7-1
4-5	6-7	6-4	5-6	7-4	5-7	4-8

(b) Corresponding RR timetable to the colored graph in a)

Fig. 1: Relation between proper edge coloring and RR timetable



(a) Partial proper edge coloring with 5 colors

R1	R2	R3	R4	R5
3-1	3-2	7-3	4-3	1-6
7-2	1-4	2-5	2-1	2-8
8-6	5-8	1-8	8-7	5-3
4-5	6-7	6-4	5-6	7-4

(b) Corresponding iRR timetable with 5 rounds to the colored graph in a)

Fig. 2: Relation between partial proper edge coloring and iRR timetable

3 Contribution

A major shortcoming of these neighborhood structures, however, is their inability to reduce non-isomorphic matches into the timetable, meaning that they never structurally modify the timetable. We fill this gap by proposing two different neighborhood structures: *incomplete Partial Team Swap* (iPTS) and *incomplete Partial Round Swap* (iPRS). iPTS swaps the opponents of two teams over a subset of the rounds, such that at most one uncolored edge is included in the swap for each of the two teams. iPRS takes as input a cycle alternating between uncolored edges and edges colored with one particular color, and swaps the colored for the uncolored edges. Hence, both iPTS and iPRS introduce new games into the iRR timetable. In fact, we show that our neighborhood structures are able to introduce non-isomorphic matches into the timetable, and prove solution space connectivity for one of them in case $r \leq \frac{n}{2}$. Moreover, we embed the neighborhood structures into a late acceptance hill climbing algorithm (LAHC) and test the algorithm on three sets of instances from the literature. Table 1 shows preliminary results for one of the considered instance sets, stemming from Belgian youth hockey competitions (see [2]). The LAHC with the proposed neighborhood structures is here compared to solving a straightforward integer program (IP) and using a metaheuristic proposed by [7] (GM), which is based on solving a sequence of greedy matchings and is the state of the art algorithm to construct iRR timetables. A time limit of 2 hours is used. The LAHC is seen to outperform the other two methods and produces solutions close to the lower bound (LB), thereby empirically confirming the validity of the proposed neighborhood structures.

Table 1: Preliminary results for some selected instances

				GM		LAHC	
Instance	LB	IP-UB	Gap (%)	UB	Gap (%)	UB	Gap (%)
i01	2821	**2821**	0.0	*2903*	2.82	**2821**	0.0
i02	3980	**3980**	0.0	*4025*	1.12	**3980**	0.0
i03	150381	*289065*	47.98	158058	4.86	**156747**	4.06
i04	180310	*339237*	46.85	190861	5.53	**186152**	3.14
i05	220520	*391507*	43.67	231584	4.78	**227254**	2.96
i06	175592	*301517*	41.76	187216	6.21	**181371**	3.19

The best found upper bound (UB) is marked in bold, while the worst found upper bound is italicized.

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