Branch-Price-and-Cut and Symmetry Reductions for Sports Tournament Scheduling

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1 Introduction

First-Break-Then-Schedule (FBTS, cf. Nemhauser and Trick [7]) is a method for optimizing sports tournament schedules that has gained widespread recognition in the literature for its effectiveness (e.g., [1,5,9]). The method decouples two core decisions: *(i)* for each team, whether they play home (H) or away (A) in each round, i.e., the *home-away pattern* (HAP); and *(ii)* which teams are meeting in which round, i.e., the *opponent schedule*. FBTS faces two critical challenges. First, the method relies on full enumeration of possible HAPs or considers heuristic approaches. On the one hand, full enumeration would give optimal solutions, but for a round set $\mathcal{R}$ and under no further constraints the set of HAPs $\{\mathsf{H}, \mathsf{A}\}^{\mathcal{R}}$ is exponential in $|\mathcal{R}|$, so full enumeration is only practical for small instances. On the other hand, heuristic restrictions of this set do not provide optimality guarantees. The second challenge is that the method lacks a systematic backtracking mechanism when HAP assignments turn out infeasible.

We provide an integer programming Branch-Price-and-Cut framework that implements FBTS in an effective and exact way. Rather than enumerating all possible HAPs, our framework starts with the most promising patterns and uses a variable pricing approach to generate missing ones. To organize backtracking, we integrate the Branch-and-Price approach with the traditional Benders' decomposition approach from [10]. Moreover, we apply the symmetry-handling method orbitopal fixing to handle symmetries among teams. Whereas our framework is broadly applicable to many classes of problems, in this extended abstract we focus on one particular problem, for which we present effective preliminary results.

2 Problem definition

We apply our framework to the *Constant-distance Traveling Tournament Problem* (C-TTP(3)), which aims to find a double-round robin (DRR) tournament where the total number of times that a team travels is minimized. More specifically:

- Each team plays exactly one match in every round;
- Each team plays at most three consecutive home games and at most three consecutive away games; and
- Matches (i, j) and (j, i) are not scheduled consecutively.

Urritia and Ribeiro [8] show that the objective considered is equivalent to maximizing the number of *breaks* in the tournament, i.e., where a break occurs if a team plays at home (or away) for two consecutive rounds.

To model the C-TTP(3) as a binary program, we introduce the following notation. Let n be an even number, the number of teams. Let $\mathcal{T} = \{1, \dots, n\}$ be the set of teams, $\mathcal{R} = \{1, \dots, 2n-2\}$ the set of rounds, and $\mathcal{M} = \{(i, j) : i, j \in \mathcal{T}, i \neq j\}$ the matches to be scheduled. Given match (i, j), i is said to be the home team and j the away team. The HAP of a team is denoted by elements in $\{\mathrm{H}, \mathrm{A}\}^{\mathcal{R}}$. A *HAP-set* consists of a HAP for each team, $\{\mathrm{H}, \mathrm{A}\}^{\mathcal{T}\times\mathcal{R}}$. Let $\mathcal{P} \subseteq \{\mathrm{H}, \mathrm{A}\}^{\mathcal{R}}$ be the set of patterns that contain precisely $n-1$ home and away games and at most three equal consecutive entries, and let $c_p := \sum_{r\in\mathcal{R}\setminus\{1\}} \mathbb{1}[p_{r-1} = p_r]$ be the number of breaks for HAP $p \in \mathcal{P}$. Note that $|\mathcal{P}|$ is exponential under these constraints, e.g. any concatenation of $n-1$ blocks (HA) and (AH) is contained in $\mathcal{P}$, yielding $|\mathcal{P}| \geq 2^{n-1}$. An exponentially-sized formulation for C-TTP(3) is:

$$\text{Maximize} \sum_{i\in\mathcal{T}} \sum_{p\in\mathcal{P}} c_p z_{i,p}, \tag{1}$$

$$\text{subject to} \sum_{p\in\mathcal{P}} z_{i,p} = 1 \qquad \text{for all } i \in \mathcal{T}, r \in \mathcal{R}, \tag{2}$$

$$\sum_{p\in\mathcal{P}:p_r=\mathrm{H}} z_{i,p} = h_{i,r} \qquad \text{for all } i \in \mathcal{T}, r \in \mathcal{R}, \tag{3}$$

$$\sum_{j\in\mathcal{T}:i\neq j} x_{(i,j),r} = h_{i,r} \qquad \text{for all } i \in \mathcal{T}, r \in \mathcal{R}, \tag{4}$$

$$\sum_{r\in\mathcal{R}} x_{(i,j),r} = 1 \qquad \text{for all } (i, j) \in \mathcal{M}, \tag{5}$$

$$\sum_{j\in\mathcal{T}\setminus\{i\}} (x_{(i,j),r} + x_{(j,i),r}) = 1 \qquad \text{for all } i \in \mathcal{T}, r \in \mathcal{R}, \tag{6}$$

$$\sum_{m\in\{(i,j),(j,i)\}} (x_{m,r-1} + x_{m,r}) \leq 1 \qquad \text{for all } (i, j) \in \mathcal{M}, r \in \mathcal{R} \setminus \{1\}, \tag{7}$$

$$h_{i,r} \in \{0, 1\} \qquad \text{for all } i \in \mathcal{T}, r \in \mathcal{R}, \tag{8}$$

$$x_{(i,j),r} \in \{0, 1\} \qquad \text{for all } (i, j) \in \mathcal{M}, r \in \mathcal{R}, \tag{9}$$

$$z_{i,p} \in \{0, 1\} \qquad \text{for all } i \in \mathcal{T}, p \in \mathcal{P}. \tag{10}$$

In this formulation, $z_{i,p}$ indicates whether i plays according to the HAP p, $h_{i,r}$ indicates whether i plays home in round r, and $x_{(i,j),r}$ indicates whether match (i, j) is played in round r. The objective function maximizes the overall

Table 1: Computational results for C-TTP(3) with n teams. We compare the compact formulation and Benders' (B) decomposition approach (cf. [10, Section 4.1]) that introduce additional variables for counting the breaks, with the pricing approach (P) and additional symmetry handling (S). We report the best dual bound (D), the best integer primal bound (P), and the runtime in seconds (T). A time limit of 3600 s applies.

n	Compact			B			B + P			B + P + S		
	D	P	T	D	P	T	D	P	T	P	D	T
4	17	17	< 1	17	17	< 1	17	17	< 1	17	17	< 1
6	43	43	34	43	43	23	43	43	< 1	43	43	< 1
8	80	80	74	80	80	37	80	80	< 1	80	80	< 1
10	105	134	3600	107	125	3600	124	124	17	124	124	3
12	147	199	3600	152	187	3600	180	181	3600	181	181	3036
14	198	280	3600	202	260	3600	252	252	417	252	252	14
16	257	352	3600	259	348	3600	327	327	535	327	327	297
18	322	—	3600	326	438	3600	414	420	3600	414	417	3600
20	398	—	3600	397	566	3600	520	520	52	520	520	54

number of breaks, Constraint (2) ensures that a HAP is selected for every team, Constraints (3–6) ensure that the result is a DRR tournament consistent with the HAP selection, and (7) enforces the remaining condition.

3 Contributions

We present a Branch-Price-and-Cut framework to solve a wide class of sports tournament scheduling problems and apply this to C-TTP(3) via a dual decomposition of the program above. Our restricted program consists of the h-variables, we apply variable pricing to the $z_{i,p}$-variables, and we apply separation to ensure existence of a feasible solution for the $x_{(i,j),r}$-variables. Furthermore, since the $h_{i,r}$-variables are persistent in the problem, we can apply symmetry-handling methods to those. More specifically, our contributions are the following:

(i) Regarding the pricing problem, although the number of variables $z_{i,p}$ is exponential in n, the pricing problem runs in polynomial time.

(ii) Regarding the separation problem, we follow the method in [10], where a traditional Benders' decomposition approach is employed to eliminate the x-variables. Briskorn [3] provides reasons to believe that a simple Linear Programming (LP) feasibility check for fixed $z_{i,p}$ variables is already a good heuristic to check feasibility of the generated HAP set, which allows for effective separation in many cases. If the LP relaxation fails to prove infeasibility, we use nogood cuts as a fallback, cf. [10].

(iii) Further, as solution quality for C-TTP(3) is invariant under team permutations, we apply dynamic orbitopal fixing (cf. [2,4]) with symmetry preprocessing on the first round to deal with the team symmetries.

(iv) Last, we provide an implementation via plug-ins for SCIP 10 [6]. In Table 1 we present some indicative preliminary results for C-TTP(3).

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References

1. Alarcón, F., Durán, G., Guajardo, M., Miranda, J., Muñoz, H., Ramírez, L., Ramírez, M., Sauré, D., Siebert, M., Souyris, S., Weintraub, A., Wolf-Yadlin, R., Zamorano, G.: Operations research transforms the scheduling of Chilean soccer leagues and South American world cup qualifiers. Interfaces **47**, 52–69 (2017). https://doi.org/10.1287/inte.2016.0861
2. Bendotti, P., Fouilhoux, P., Rottner, C.: Orbitopal fixing for the full (sub-)orbitope and application to the unit commitment problem. Mathematical Programming **186**(1–2), 337–372 (Dec 2019). https://doi.org/10.1007/s10107-019-01457-1
3. Briskorn, D.: Feasibility of home–away-pattern sets for round robin tournaments. Operations Research Letters **36**(3), 283–284 (May 2008). https://doi.org/10.1016/j.orl.2007.09.009
4. van Doornmalen, J., Hojny, C.: A unified framework for symmetry handling. Mathematical Programming **212**(1–2), 217–271 (Jun 2024). https://doi.org/10.1007/s10107-024-02102-2
5. Durán, G., Gutiérrez, F., Guajardo, M., Marenco, J., Sauré, D., Zamorano, G.: Scheduling the main professional football league of Argentina. INFORMS Journal on Applied Analytics **51**, 361–372 (2021). https://doi.org/10.1287/inte.2021.1088
6. Hojny, C., Besançon, M., Bestuzheva, K., Borst, S., Chmiela, A., Dionísio, J., Eifler, L., Ghannam, M., Gleixner, A., Göß, A., Hoen, A., van der Hulst, R., Kamp, D., Koch, T., Kofler, K., Lentz, J., Maher, S.J., Mexi, G., Mühmer, E., Pfetsch, M.E., Pokutta, S., Serrano, F., Shinano, Y., Turner, M., Vigerske, S., Walter, M., Weninger, D., Xu, L.: The SCIP Optimization Suite 10.0. Technical report, Optimization Online (November 2025), https://optimization-online.org/2025/11/the-scip-optimization-suite-10-0/
7. Nemhauser, G.L., Trick, M.A.: Scheduling a major college basketball conference. Oper. Res. **46**, 1–8 (1998). https://doi.org/10.1287/opre.46.1.1
8. Urrutia, S., Ribeiro, C.C.: Maximizing breaks and bounding solutions to the mirrored traveling tournament problem. Discrete Appl. Math. **154**, 1932–1938 (2006). https://doi.org/10.1016/j.dam.2006.03.030
9. Van Bulck, D., Goossens, D.: First-break-heuristically-schedule: Constructing highly-constrained sports timetables. Operations Research Letters **51**, 326–331 (2023). https://doi.org/https://doi.org/10.1016/j.orl.2023.03.014
10. Van Bulck, D., Goossens, D.: A traditional Benders' approach to sports timetabling. European Journal of Operational Research **307**(2), 813–826 (Jun 2023). https://doi.org/10.1016/j.ejor.2022.10.044