A Hyperbolic Multi-Strategy Search Framework for Flexible Job Shop Scheduling

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Abstract. The Flexible Job Shop Scheduling Problem (FJSP) requires simultaneous machine assignment and operation sequencing decisions, resulting in a large combinatorial search space. Existing swarm-based scheduling approaches often rely on homogeneous initialization and fixed search behaviours, which may limit search diversity and reduce search effectiveness. This paper proposes a hyperbolic multi-strategy search framework for single-objective FJSP. The framework employs a nonlinear hyperbolic representation to improve population diversity and search-space coverage, together with adaptive operator switching among cosine exploitation, hyperbolic intensification, and Lévy-flight exploration. Candidate solutions are transformed into feasible schedules using random-key encoding and earliest finishing time (EFT) decoding. Preliminary ablation and comparative studies on representative Brandimarte benchmark instances demonstrate competitive makespan performance across different scheduling scales.

Keywords: Flexible Job Shop Scheduling · Scheduling Optimization · Adaptive Search Dynamics · Swarm Optimization

1 Introduction

The Flexible Job Shop Scheduling Problem (FJSP) is a widely studied production scheduling problem due to its practical importance in modern manufacturing systems. The simultaneous determination of machine assignments and operation sequences creates a large combinatorial search space with strong interactions between routing and sequencing decisions, resulting in an irregular search landscape and numerous local optima [3, 1]. Consequently, effective diversification and adaptive search regulation are essential for generating high-quality schedules.

Swarm-based metaheuristics have demonstrated competitive performance for FJSP due to their ability to explore complex scheduling spaces. However, generating high-quality schedules for medium- and large-scale instances remains challenging, motivating the development of more effective search mechanisms. Search diversity and the balance between exploration and exploitation play a critical role in the performance of population-based optimization algorithms. Homogeneous population initialization may limit the diversity of candidate schedules,

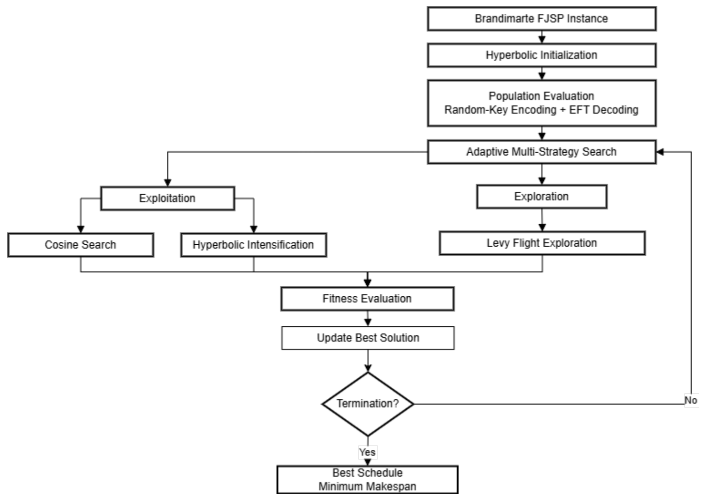

Fig. 1. Overview of the proposed hyperbolic multi-strategy search framework.

while fixed search behaviours may reduce the ability to adapt search intensity throughout the optimization process. These limitations motivate the investigation of nonlinear search representations and adaptive search strategies for FJSP optimization. To address these challenges, this study proposes a hyperbolic multi-strategy search framework that combines nonlinear hyperbolic representation, adaptive search regulation, and EFT-based schedule construction. The main contributions are: (1) a nonlinear hyperbolic representation to improve population diversity and search-space coverage; (2) an adaptive multi-strategy search framework incorporating cosine exploitation, hyperbolic intensification, and Lévy-flight exploration; and (3) a random-key and EFT-based schedule construction mechanism for generating feasible FJSP solutions.

In this study, the objective is to minimize the makespan while satisfying operation precedence, machine capacity, and machine assignment constraints. The objective function is defined as:

$$C_{\max} = \max_{i \in J} C_i \tag{1}$$

where C_i denotes the completion time of job i.

2 Hyperbolic Multi-Strategy Search Framework

The proposed framework addresses the FJSP by integrating heterogeneous initialization, adaptive search regulation, and schedule construction mechanisms.

Figure 1 presents the proposed framework. Hyperbolic initialization (Eq. 2) is used to improve population diversity and search-space coverage. During optimization, the adaptive parameter in Eq. 3 regulates cosine exploitation, hy-

Table 1. Ablation study results on representative Brandimarte benchmark instances.

Instance	AH-SCSA		AH-SCSA$_2$		AH-SCSA$_1$	
	Best	Avg	Best	Avg	Best	Avg
Mk01	39	40.0	39	41.7	39	41.9
Mk05	174	174.0	187	191.7	186	191.1
Mk08	523	523.0	531	548.0	534	548.4

perbolic intensification, and Lévy-flight exploration. Candidate solutions are decoded using random-key encoding and EFT decoding, and evaluated using the makespan objective in Eq. 1.

For exploitation, candidate solutions are refined using the original cosine-based search mechanism inherited from SCSA together with the proposed hyperbolic intensification operator(Eq. (5)). Lévy-flight perturbation is employed to preserve diversification and reduce stagnation Eq. (6).

$$X_{i,j} = lb_j + \frac{\tanh(\alpha \cdot rand)}{\tanh(\alpha)} (ub_j - lb_j) \tag{2}$$

where α controls the dispersion intensity, $rand \in [0, 1]$, and lb_j and ub_j denote the lower and upper search bounds, respectively.

$$R(t) = SM \left(1 - \frac{2t}{T + t}\right) \tag{3}$$

$$X_i^{t+1} = X_{\text{best}} - R(t)\, r \left(X_{\text{best}} - X_i^t\right) \cos(\theta) \tag{4}$$

$$X_i^{t+1} = X_i^t + \tanh(R(t))\; r_1 \left(2r_2 X_{\text{best}} - X_i^t\right) \tag{5}$$

$$X_i^{t+1} = X_i^t + \lambda, \mathrm{Levy}(\beta) \tag{6}$$

where λ controls the step size and Levy(β) denotes a Lévy-distributed random walk.

3 Preliminary Experimental Results

The proposed framework was evaluated on three representative Brandimarte instances: Mk01, Mk05, and Mk08, representing small-, medium-, and large-scale scheduling problems. These instances enable the evaluation of solution quality and scalability across different scheduling complexities.

Table 1 compares three variants of the proposed framework. *AH-SCSA*$_1$ represents the baseline SCSA with only the discretization mechanism, while *AH-SCSA*$_2$ additionally incorporates heterogeneous initialization. The complete *AH-SCSA* further includes the proposed adaptive search operators. This comparison evaluates the contribution of population diversity and adaptive search mechanisms to FJSP scheduling performance.

Table 2. Comparative study results on representative Brandimarte benchmark instances.

Instance	AH-SCSA		SS-GWO[6]		DBO+SA [5]		AOOA [7]		ISCSO [2]	
	Best	Avg	Best	Avg	Best	Avg	Best	Avg	Best	Avg
Mk01	**39**	40.0	**39**	–	40	41.3	45	–	42	42.6
Mk05	**174**	**174.0**	179	–	175	177.6	197	–	179	184.3
Mk08	**523**	523.0	541	–	**523**	526.4	**523**	–	**523**	523.1

Additional comparisons with representative swarm-based and recent FJSP optimization methods are included. The preliminary results shown in table 2 indicate that the proposed framework achieves competitive makespan values across different problem scales. These results indicate that the proposed hyperbolic multi-strategy search framework is capable of producing high-quality schedules across different problem scales.

4 Conclusion

This paper proposed a hyperbolic multi-strategy search framework for the single-objective Flexible Job Shop Scheduling Problem. The framework combines hyperbolic initialization, cosine exploitation, hyperbolic intensification, Lévy-flight exploration, and EFT-based schedule construction. Preliminary results on selected Brandimarte benchmark instances demonstrate competitive scheduling performance and highlight the potential of combining adaptive search regulation with nonlinear search representations for scheduling optimization.

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