The Incomplete Traveling Tournament Problem

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1 Introduction

In a double round robin sports timetable, each team plays every other team once at home and once away. The Traveling Tournament Problem (TTP) seeks to construct such a timetable while minimizing the total travel distance of all teams (see also [2, 3]). It is one of the best studied sports timetabling problems, with research focusing on the development of lower bounds (see, e.g., [9]), as well as heuristic (e.g., [4]), exact (e.g., [8]), and approximation algorithms (e.g., [10]). In fact, the TTP has featured at PATAT on several occasions (e.g., [5, 6]).

In an incomplete round robin timetable, all teams play the same number of games, but not necessarily against the same subset of opponents. The choice of what games to include is left to the organizer. This format is rapidly gaining popularity, with a prominent example being the group stage of the UEFA Champions League. Applications have also emerged in non-professional youth competitions (e.g., [1, 7]).

This extended abstract discusses the incomplete Traveling Tournament Problem (iTTP), which introduces the TTP into the realm of incomplete round robin timetabling where not all teams play each other an equal number of times (e.g., like in the NBA)[1]. Given an even number of teams and an even number of rounds, with the number of rounds less than the number of teams, as well as the pairwise distances between the teams' home venues, a feasible solution to the iTTP is an incomplete round robin timetable such that:

(C1) each team plays exactly one game per round,
(C2) each team plays the same number of home and away games,
(C3) no team plays more than three home or away games in a row, and
(C4) each team meets every other team *at most* once.

The objective in the iTTP is to minimize the overall distance travelled by all teams. We assume that teams that consecutively play away travel directly between the opponent's venues, and that they start and end the season at home. As will be shown in this talk, the iTTP is $\mathcal{NP}$-hard even when restricted to only two rounds (making that (C3) does not apply). We introduce several new benchmark instances which are based on those for the classic TTP and which are available at the website of RobinX[2].

[1] An extended version is available at https://arxiv.org/abs/2603.19754.
[2] See https://robinxval.ugent.be/RobinX/travelRepo.php.

2 Lower Bounds

A first attempt to obtain tight lower bounds for the iTTP was made by computing an analogue of the independent lower bound (ILB; see also [9]), which computes the minimal travel distance for each team independently from all the others. In the iTTP, the ILB corresponds to solving for each team an instance of the prize-collecting vehicle routing problem, in which some customers may be skipped as long as a given total prize is collected. While the ILB is known to provide tight bounds for the classical TTP, it turns out to be surprisingly weak for the iTTP. The main reason is that it does not prevent two teams from visiting each other when they are mutual nearest neighbours, which is forbidden in the iTTP by *(C4)*. At the same time, remote teams that are not among the nearest neighbours of any other team are possibly never visited (violating *(C2)* for this remote team). To address these issues, we propose several extensions of the ILB and refer to the resulting lower bound as the Dependent Lower Bound (DLB). The DLB constructs the set of roadtrips of teams, ensuring no pair of teams meets more than once, without considering the rounds in which games are scheduled.

3 Solution Strategies

Even the smallest of the proposed benchmark instances for the iTTP seem non-trivial to solve. We experimented with a straightforward Integer Programming (IP) formulation as well as a more advanced formulation that contains a variable for each possible road trip. Moreover, we propose a hill climbing heuristic that proceeds in two phases. First, it determines the teams' home-game rounds, using a simple neighbourhood structure for which we prove that it connects the search space of all possible home-game round assignments. Given such predetermined home-game rounds, we then construct a timetable by sequentially solving bipartite matching problems (for the second step, see also [7]).

4 Computational Results

Table 1 provides computational results for two classes of the proposed benchmark instances: the National League (NL) of Major League Baseball and the National Football League (NFL). Both classes stem from the original TTP: the number before the dash in the instance name indicates the number of teams in the instance, while the number after the dash indicates the number of rounds.

The first two columns show that the ILB is indeed quite weak for the iTTP: DLB significantly improves upon it on all considered problem instances. Our proposed bound DLB is also much stronger than the LP relaxation of a naive and a trip-based IP formulation (columns LP).

Regarding the best found solutions, we see that the IP formulation with trip variables performs quite well on the NL instances. At the same time, the number of trip variables explodes for the NFL problem instances, causing this model to

Table 1. Computational experiments. IP formulations are run with 48 hours of computation time, while the matching heuristic runs on average for about 10 minutes. A '-' means the IP model could either not be fit into 64GB of memory, or the solver ran out of memory before finding a feasible solution.

			Naive IP		Trip IP		Matching Heuristic	
Instance	ILB	DLB	LP	UB	LP	UB	Best UB	Avg UB
NL16-4	15273	24464	9002	25200	22696	25076	26112	28517
NL16-8	36130	57263	13598	70448	52905	62125	66763	73147
NL16-12	55568	92580	20760	153583	91346	110037	120310	131165
NFL32-8	47506	70158	10275	115763	64912	-	85201	91932
NFL32-16	113319	175051	19857	362382	169263	-	235752	255240
NFL32-24	191083	297305	36216	651580	-	-	443518	467803

run out of memory. In these cases, the matching heuristic performs much better than the naive formulation, yet the gap with the lower bounds remains large. This once more demonstrates that the iTTP problem instances are non-trivial to solve, even though much fewer games have to be planned per team compared to the classic TTP.

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