Mathematical Optimization of the Rail Infrastructure Planning Timetable

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Abstract. The long-term timetabling of railway renewal and maintenance is a complex and large-scale problem with significant economical and societal implications. We present a mixed-integer programming model for solving the Danish Rail Renewal and Maintenance Problem (DRRMP), considering the trade-offs between renewal, maintenance and their impact on traffic disturbance.
We consider a time horizon of between 60 and 120 years and the entire Danish railway network. The proposed model incorporates realistic operational constraints provided by Banedanmark, and is evaluated on data provided by Banedanmark, to test its applicability on large-scale infrastructure planning problems.

Keywords: Transport · Strategic planning · Mathematical Programming.

1 The Danish Rail Renewal and Maintenance Problem

Banedanmark (BDK), the Danish railway infrastructure manager, is responsible for producing an annual financial budget that includes a long-term timetable of renewal and maintenance activities. To produce a realistic and reliable budget, a renewal and maintenance timetable must be constructed. This is an extremely intricate and costly process, as the Danish railway infrastructure consists of thousands of rail-segments, sleepers, ballast and switches, spanning over 3000 km. Further details about the network is provided in figure 1. Currently, BDK relies on legacy software to construct the renewal timetable,

Fig. 1. Map of the Danish railway network

and a recent market survey failed to identify suitable alternative software solutions. For this reason, we introduce a complex mathematical model for solving the Danish Rail Renewal and Maintenance Problem (DRRMP) on a strategic level with a time horizon spanning from 60 on 120 years. To the best of our knowledge, no previous studies have considered the same planning horizon on a railway network of this scale. However, related work have addressed similar railway scheduling and planning problems. [1] address the same maintenance planning problem on the Swedish railways and uses a using a matheuristic to construct a day-to-day plan with a time horizon of 10 days. [3] considers an annual planning of 600 fixed maintenance projects of the Dutch railway system using a Hybrid Greedy-ES algorithm, and finally . Furthermore the DRRMP differentiate from the problem posted by [3] considers scheduling predetermined project requests on a much smaller time scale, whereas the DRRMP model considered the creation and scheduling of renewal and maintenance projects on a much larger scale.

1.1 Objective

The purpose of the DRRMP model is to provide BDK with a new state-of-the-art method to construct renewal and maintenance timetables. This model evaluates the trade-offs between renewing and maintaining the railway assets, along with the socioeconomic impact on the system. To this end , the model considers 3 intertwined sub-objectives *Maintenance cost*, *Renewal Cost*, and *Traffic Hindrance Cost*. As all three components are expressed in monetary terms (million DKK), they can be aggregated directly into the single linear objective function shown in (1). This enables the model to balance the direct costs of maintenance and renewal activities against the socioeconomic costs associated with train delays.

Maintenance cost: The cost of maintaining a given asset not only depends on the assets age - generally the cost increases as the asset gets older - but also on the amount of traffic on the asset. Assets located on heavily trafficked sections require more frequent maintenance. The price of maintaining an asset is often cheaper than renewing it. However an asset can only be maintained a certain amount of times before renewal is required.

Renewal cost: The cost of renewing assets depends on the combination of asset renewals. As briefly described earlier, an asset is one of four items, *rails*, *sleepers*, *ballast* or *switches*, where the first three are physically located within the track structure. For this reason, the cost of renewing a combination of assets may sometimes be be cheaper than the sum of the individual renewals.

Traffic Hindrance Cost (THC): The THC represents the socioeconomic impact of the renewal and maintenance timetable. As railway assets deteriorate, the likelihood of speed restrictions and operational disruptions increases, leading to train delays. These delays impose costs on society by increasing travel times for passengers and delaying the transport of goods. Consequently, THC quantifies the socioeconomic consequences associated with reduced infrastructure performance and the resulting train delays.

1.2 Instance

As mentioned, the model will be solving a real-would instance of planning renewal and maintenance over the next 60 to 120 years for the entire Danish railway network, a system consisting of over 3000 km of rails, sleepers, and ballast. BDK has divided the different assets into segments, with the shortest segments being only 10 meters, resulting in the system having more than 24.000 assets. As some assets are physically located on top of each other, they will be bundled into collections called *asset collections*, such that a single asset collection consists of *rails* (steel tracks guiding the train), *sleepers* (transverse supports holding the rails in place) and *ballast* (crushed stone providing stability and drainage) or just *switches and crossings* (track components enabling trains to change from one track to another) . This results in over 9.000 asset collections. The decision is therefore not whether to renew or maintain individual assets, but which renewal and maintenance *activity* should be done on the asset collection. A renewal or maintenance activity may involve a single asset or a combination of assets within the same collection. Furthermore, BDK groups the renewal and maintenance into projects, as they want to ensure some assets are renewed or maintained at the same time as others, and to ensure a somewhat consistent amount of activities planned throughout the year.

The full-scale timetable instance results in a highly combinatorial problem with millions of possible activity assignments. The model must capture interactions between assets, project bundling, and the probabilistic impact of asset deterioration on train delays, which makes the formulation both large and computationally challenging.

1.3 Model Overview

The DRRMP formulation incorporates the operational constraints considered by BDK when constructing a renewal and maintenance timetable. The most important constraints are summarized in Table 1: Important constraints. In addition to replicating the functionality of the current legacy software, the model also introduces several additional constraints that allow planners to better control the structure of renewal projects, listed in 1: Additional constraints.

The core decision variable of the model is the binary variable $X^{t,p}_{ac,k}$. The variable equals 1 iff renewal or maintenance activity k is performed on asset collection ac in time period t as part of project p, and 0 otherwise. These decisions determine the renewal and maintenance schedule of all asset collections over the timetable horizon. From the selected activities, the model derives the resulting asset conditions, project composition and associated costs.

The objective is to minimize the total cost of renewal, maintenance, and traffic hindrance over the timetable horizon:

$$\min \sum_{t} (C^t_{renewal} + C^t_{maintenance} + C^t_{THC}) \tag{1}$$

The resulting formulation is a large-scale mixed-integer program, which when instantiated on a single line section contains over 300.000 variables and 6 million

	Important constraints
Activity limit	At most one activity can be performed on an asset collection in a year.
Renewal pause	Renewing an asset incurs a 10 year renewal pause on all assets on the same line.
Max train delay	An upper bound on the number of trains that are delayed each year.
Max activity volume	An upper bound on the amount of each activity performed each year.
	Additional constraints
Horizontal bundling	Renewal activities must be grouped into contiguous track segments with a minimum project length.
SVAM	Modeling the number of track-closing-days when work is being conducted.
Project stability	The number of projects may only change by a given amount from year to year.
Renewal budget	The renewal cost may not exceed the renewal budget each year.
Maintenance budget	The maintenance cost may not exceed the maintenance budget each year.

Table 1. Key constraints included in the DRRMP model. Not all constraints are included in the table.

constraints. Scaling to the full Danish railway network, consisting of 50 line sections, results in more than 60 million variables and 130 million constraints. This leads to a highly memory-intensive model that exceeds the capabilities of standard personal computers. Consequently, all experiments are conducted on the high-performance computing cluster [2] at the Technical University of Denmark.

2 A brief conclusion

As the project is still in progress, results from solving the model on the entire network and smaller subparts will be presented along with the conclusions drawn.

References

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